# II. *The Calculus of Chemical Operations; being a Method for the Investigation, by means of Symbols, of the Laws of the Distribution of Weight in Chemical Change.—*

## Part II. *On the Analysis of Chemical Events.*

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> "*The Observer is not he who merely sees the thing which is before his eyes, but he who sees what parts that thing is composed of.*"—J. Stuart Mill.

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### Introduction.—ON THE LAW OF EVEN NUMBERS.

The first part of this Calculus was devoted to the construction of those rudimentary tools of analytical investigation termed Chemical Symbols. I have there given expression, by a system of arbitrary signs, to certain mental conceptions and combinations of conceptions which enter into exact chemical inquiries. This involves an analysis of those conceptions. But something more, too, is there effected. For the study of these questions necessitates the reconstruction (to a certain extent) of the fundamental ideas of the science and, especially, the reconsideration of a problem supposed to have been, long since, finally determined, namely, the constitution of the units of ponderable matter, of which I have given a new theoretical analysis. I shall not attempt to give any summary of these results, which, in my previous Memoir, have been discussed as briefly as is consistent with clearness. At the same time I should observe that the following pages can only be intelligible to those who have already made themselves acquainted with the principles of this Calculus, and to such alone they are addressed. There is, however, a point of fundamental importance which as yet has been only incidentally touched, namely, the origin of the hypothesis that the unit of hydrogen is an "undistributed weight," which is the keystone of the system here adopted, and the reasons by which that hypothesis is justified, on which it is desirable, before proceeding further, to offer a somewhat fuller explanation. The following slight outline of the treatment of the subject pursued in this Calculus is given merely with the view of introducing these questions.

For the effective consideration of the chemical properties of matter it is necessary to refer these properties to a common standard of comparison. Our first step, therefore, was the definition of the "Unit of ponderable matter" (I. Section I. (10)). In the selection of this unit we are guided by the same principles as those on which we select the unit of length, the unit of weight, the unit of heat, our choice being in all such cases determined by what is convenient for the special purpose in view. Now the objects of our study are the chemical nature and transformations of gaseous matter. If, therefore, we wish to reason with impartiality, we must compare the properties of equal volumes of gases existing under the same conditions of temperature and pressure, for no reason can be assigned for comparing unequal volumes. All gases must be treated alike.

We are thus led to the notion of the "unit of space," a peculiar and essential feature of this Calculus. This "unit of space" is the empty measure in which we measure out the "units of matter." The measure chosen is 1000 cub. centims.; and were we to measure out the volumes of gases in this standard measure at a temperature of 0° C. and a pressure of 760 millims., we should be in possession of the "units of matter." We shall consider this estimate to have been, in some way, made.

Here we are met by the preliminary difficulty that this estimate involves numerous assumptions, so that our results are by no means conclusive. Even in the case of actual gases and vapours we cannot determine our units without, at least, assuming the truth of the law of MARIOTTE and of the recognized variation of volume due to changes of temperature. But the greater number of chemical substances do not exist for us as gases at all. Here we are driven to select as the "unit of matter" that quantity of matter which we infer, from probable reasoning of various kinds, would occupy our measure, in the gaseous state, under certain normal conditions, if only we could thus estimate it. Hence undoubtedly our results are liable to be affected with very serious errors. So that it may be truly said that the theory of chemistry refers rather to an ideal world created by the reason of man out of the actual world than to the actual world itself. There is, however, a wide interval between the ideal creations of the reason and the figments of the imagination. In the long run the former correspond to realities, and the comparison of the results as to the densities of gases which have actually been anticipated by this mode of probable reasoning with those subsequently arrived at by conclusive experiments prove us to be on the right track.

If it be asked why we do not, as is usually done, select a "molecule" as our unit, we reply, not that the molecular hypothesis is untrue (this would be going beyond the mark), but that it is unnecessary. We do not object to it, but we do not use it, for we do not want it for our present purpose. Having thus obtained the "unit of matter," the inquiry before us is, by what operations are these units of matter made? We begin by endeavouring to answer another question, namely, of what are they made? To this but one rational reply can be given. The units of matter are made of the matter of one another. But this is not all that can be said, for we may proceed to ask how, in particular cases, these units are thus made up.

The matter of two units of water, 2C, is made up of the matter of two units of hydrogen, 2A, and one unit of oxygen, D, and is therefore identical with that matter. We may state this identity in an equation, thus—

$$2C=2A+D;$$

whence

$$D=2C-2A,$$

$$=2(C-A).$$

Now we give a complete definition of the unit of oxygen, D, when we say that the matter of a unit of oxygen is identical with the matter of two units of water without the matter of two units of hydrogen; but we may further, from the last equation, assert that the matter of a unit of hydrogen, A, is a part of the matter of a unit of water, C. If, then, in the above equation, making this assumption, we put $C=B+A$, we have $D=2B$.

Further, assuming the matter of two units of water to be identical with the matter of a unit of hydrogen and the matter of a unit of binoxide of hydrogen*, and putting E as the matter of a unit of binoxide of hydrogen, we have

$$2C=A+E,$$

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* The vapour-density of binoxide of hydrogen has not been experimentally determined. If this vapour-density should prove to be other than that here given, we should be compelled to take a different view of the constitution of water; but, from all we know, this is a highly improbable contingency.

whence, substituting for C the value $A+B$, we have

$$2A+2B=A+E;$$

and $E=A+2B$, giving, as the constituents of the unit of matter considered:—

| | | | |
|---|---|---|---|
| Unit of Hydrogen | $A$ | weighing | ·089 grm. |
| „ Oxygen | $2B$ | „ | 1·430 „ |
| „ Water | $A+B$ | „ | ·805 „ |
| „ Binoxide of hydrogen | $A+2B$ | „ | 1·520 „ |

We cannot, from the evidence before us, assign any constitution to the bits of matter A and B, nor are we able to make any assertion about these bits of matter, except that the bit of matter A weighs ·089 grm., and that the bit of matter B weighs ·715 grm.

A bit of matter as to which our knowledge is thus limited is what is here termed a "simple" or "undistributed weight." These terms do not refer to the constitution of the bits of matter A and B, as to which we have no means of forming an opinion, but to a very different thing, namely, our knowledge of that constitution, which is confined within narrow bounds determined by our powers of observation and experiment.

The groups of letters $A$, $2B$, $A+B$, $A+2B$, although truly symbols or signs of the matter of which the units of hydrogen, oxygen, water, and binoxide of hydrogen consist, are not the symbols of those units of matter themselves. A symbol must distinguish the thing symbolized from other things, but these expressions do not thus distinguish those units of matter. Thus the expression $A+2B$ indicates (to one who knows the meaning here given to the letters) the matter of which the unit of binoxide of hydrogen is constituted; but it indicates to him several other things besides, namely, the matter of a unit of hydrogen, $A$, and the matter of a unit of oxygen, $2B$, and also the matter of a unit of water, $A+B$, and the matter of the simple weight $B$, and also the matter of the unit of hydrogen, $A$, and of two simple weights, $B$ and $B$. So that when such an expression is presented to us (if no more be said) we cannot tell to which of these different objects the expression refers. The expression $A+2B$ indicates something common to all these objects, but does not indicate any one object specially. How, then, are we to frame such a distinctive symbol? This question has been fully considered in the first part of this Calculus, to which I must refer the reader for information. I will here only make one remark. The chemical symbol of a unit of matter, as constructed on the principles of this Calculus, is an analytical expression which indicates to one acquainted with those principles that special combination of operations by which, in the processes of chemical change, the unit has been made up. These operations are known to us only through their results, and are defined by those results; indeed no other knowledge of them is practicable; but this is sufficient for our purpose, which is the comparison of those results.

That operation which I have here termed a chemical operation, and defined as an operation performed upon the unit of space, of which the result is "a weight"

(I. Sec. II. (1)), is an operation with one variety of which chemists are familiar under the name "chemical combination." I have not employed this language, for the terms are by no means coextensive; and we should be led into unnecessary difficulties by restricting "a chemical operation" to that particular kind of "chemical operation" implied in the term "combination" with all the ideas which have grown up around it. If, however, we are willing to discard material images, and consider "combination" in a more general and abstract sense, the term will work. Let $x, y, z \ldots$ be the symbols of the operations of chemically combining the "weights" or bits of matter, A, B, C, . . ., at 0° and 750 millims. pressure, then $x, y, z \ldots$ are termed the chemical symbols of A, B, C . . . respectively at that temperature and pressure. Also let $xy$ be the symbol of the "combination" of A and B, and $xyz \ldots$ the symbol of the "combination" of A and B and C . . ., then $xxx \ldots$ is the symbol of the "combination" of A and A and A . . . Further let $z$ be the symbol of the "weight" or matter contained in an empty unit of space, then $zz$ (or $z^2$) is the symbol of the combination of the "weights" (or matter) contained in two empty units of space, and $z^p$ is the symbol of the combination of the "weights" in $p$ empty units of space. But the "weights" in two empty units of space are collectively identical with the "weight" in one empty unit of space, being in both cases "no weight," and the weights in $p$ empty units of space are collectively identical with the "weight" in one empty unit; we have, therefore, $z^p=z$, and generally $xy \ldots z^p=xy \ldots$ Now among the symbols of number we have one symbol, and one symbol alone, which satisfies the condition satisfied by the symbol $z$, namely, the symbol 1. If, therefore, we put 1 as the symbol of the "weight" contained in an empty unit of space, and work with this symbol (as a factor) in the algebra of chemistry, according to precisely the same rules as in general algebra we work with the numerical symbol 1, we shall never be led into error. My object in these remarks is to point out the intimate connexion which subsists between the principles of this Calculus and those fundamental ideas which have been developed by the requirements of the science*. I shall not pursue the subject, as it has been fully treated in Part I. Sec. III.

The symbols of the units of hydrogen, oxygen, water, and binoxide of hydrogen, as thus expressed, are:—

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* The objects of chemistry, considered as an art and as a science, were defined by the illustrious STAHL in the following words:—"Chymia alias Alchymia et Spagirica, est ars corpora vel mixta vel composita vel aggregata etiam in principia sua resolvendi aut ex principiis in talia combinandi.

"Subjectum ejus sunt omnia mixta et composita quæ resolubilia et combinabilia. Objectum est ipsa resolutio et combinatio, seu corruptio et generatio."—STAHL, *Fundamenta Chymiæ*, Norimbergæ MDCCXXIII.

The "resolubilia" and "combinabilia" of STAHL correspond to what is here termed ponderable matter [I. Sec. I. Def. (1)], while the "ipsa combinatio" and "ipsa corruptio" represent, in his order of ideas, what is here termed a "chemical operation" [I. Sec. II. Def. (1)].

| | |
|---|---|
| Unit of Hydrogen | $\alpha$, |
| „ Oxygen | $\xi^2$, |
| „ Water | $\alpha\xi$, |
| „ Binoxide of hydrogen | $\alpha\xi^2$, |

$\alpha$ being the symbol of the "weight" A, and $\xi$ the symbol of the "weight" B.

These symbols involve no hypothesis whatever except the accuracy of the data from which they are deduced; and were we ignorant of the existence of the elements chlorine, iodine, bromine, nitrogen, phosphorus, . . ., and the other elements (hydrogen excepted) of the class termed by LAURENT "dyad elements"*, it would be in our power, proceeding on these principles, to construct a perfectly unhypothetical symbolic system (which would be open to no difference of opinion) to represent the units of matter.

Now, however (retaining A as indicating the matter of a unit of hydrogen), let E be the matter of a unit of hydrochloric acid and C the matter of a unit of chlorine. We have then

$$2E=A+C,$$

and

$$C=2E-A;$$

the only inference to be necessarily drawn from this equation is that the matter of a unit of hydrogen is contained in (or is a part of) the matter of two units of hydrochloric acid.

Assuming each of the two units of hydrochloric acid to be similarly constituted (for we cannot draw any distinction between them without an assumption, which is totally unnecessary), this condition may be satisfied in two ways. I. A unit of hydrogen may be contained in one unit of hydrochloric acid. II. A unit of hydrogen may be contained in two units of hydrochloric acid and not contained in one unit of hydrochloric acid.

Considering Case I., let $E=A+K$, $C=A+2K$, and we have as the constituents of the units of hydrogen, chlorine, hydrochloric acid,

| | | | |
|---|---|---|---|
| Unit of Hydrogen | $A$ | weighing | ·089 grm., |
| „ Chlorine | $A+2K$ | „ | 3·173 grms., |
| „ Hydrochloric acid | $A+K$ | „ | 1·631 grm., |

the weight of the bit of matter K being 1·542 grm.

This view corresponds to the hypothesis that the unit of hydrogen is a simple weight.

| | |
|---|---|
| Symbol of Hydrogen | $\alpha$, |
| „ Chlorine | $\alpha\chi^2$, |
| „ Hydrochloric acid | $\alpha\chi$. |

In Case II. half a unit of hydrogen (and no more or less) must be contained in a unit of hydrochloric acid. Putting, then, $E=\frac{A}{2}+D$, we have $C=2D$. The constituents of

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* LAURENT, 'Chemical Method,' Cavendish Society, 1855.

the several units being

| | | | |
|---|---|---|---|
| Unit of Hydrogen . . . . . . | $2\times\frac{A}{2}$, | weighing | 0·089 grm., |
| „ Chlorine . . . . . . | 2D | „ | 3·173 grms., |
| „ Hydrochloric acid . . | $\frac{A}{2}+D$ | „ | 1·631 grm., |

the weight in grammes of the bits of matter $\frac{A}{2}$ and D being respectively 0·044 grm. and 1·586 grm. This result corresponds to the view now generally received by chemists as to the constitution of these units, according to which the matter of the unit of hydrogen is regarded as constituted of two bits of matter in all respects identical. Were we to construct the symbols of these units on the principles of this Calculus, we should arrive at the following system, which corresponds to the assumption that the unit of hydrogen is constituted of two simple weights:—

| | | | |
|---|---|---|---|
| Symbol of the unit of Hydrogen . . . . | $a^2$, | weighing | 0·089 grm. |
| „ „ Chlorine . . . . | $c^2$ | „ | 3·173 grms. |
| „ „ Hydrochloric acid . | $ac$ | „ | 1·631 grm. |

It is important to observe that there are only two such symbolic systems*, which,

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* Besides the systems originating in these two hypotheses respectively, there is also the general or indeterminate system comprehending both, in which we have

| | Symbol. |
|---|---|
| Hydrogen . . . . . . . . . . | $\alpha'\varpi'^2$ |
| Chlorine . . . . . . . . . . . | $\alpha'\chi'^2$ |
| Hydrochloric acid . . . . . . | $\alpha'\chi'\varpi'$ |
| Nitrogen . . . . . . . . . . | $\alpha'\nu'^2$ |
| Ammonia . . . . . . . . . . | $\alpha'^2\nu'\varpi'^3$ |
| . . . . . . . . . | . |
| . . . . . . . . . | . |

If in this system of symbols we put $\varpi'=1$, it becomes the system on hypothesis $a$; if we put $\alpha'=1$, it becomes the system on hypothesis $a^2$. We may consider the system to originate in the following manner. Taking the equation

$$2E=A+C,$$

let X be the matter common to A and C, and Y and Z the other constituents of A and C respectively, so that

$$A=X+Y,$$

$$C=X+Z;$$

we have then

$$2E=(X+Y)+(X+Z)$$

and

$$E=X+\frac{Y+Z}{2}.$$

Since Y and Z have no common part, $\frac{Y}{2}$ must be a "simple weight," and $\frac{Z}{2}$ also a simple weight, whence we have the units of hydrogen, chlorine, and hydrochloric acid constituted as follows:—

without irrelevant assumptions, can possibly be constructed—the system to the base $\alpha$[*], based on the hypothesis that in chemical transformations the unit of hydrogen is an "undistributed" or "simple weight," and the system to the base $a^2$, based on the assumption that the unit of hydrogen is constituted of two simple weights into which that unit is distributed in those transformations. These systems are mutually exclusive, and cannot both be true. Now we are not in a position to assert that the one of these systems is true and the other false. If so, one system would be applicable and the other inapplicable to the facts, and in the construction of our system we should get rid of hypothesis altogether. It is rarely, however, indeed that in any case such a scientific construction is practicable. But failing this, and admitting the inadequacy of our information, we may still ask, can we give a reasonable preference to the one or the other system? Even this might be out of our power. To what extent, too, is this preference to be carried? These are questions of probable reasoning which must be left to the judgment of individuals; but if we are to assume these questions to be decided before we begin to consider them, we may as well not consider them at all.

I may first observe that the two systems are not at the outset placed upon a precisely equal footing; for it is certain that in a very large proportion of chemical events the

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| | |
|---|---|
| Unit of Hydrogen . . . . . . | $X+2\left(\frac{Y}{2}\right)$, |
| „ Chlorine . . . . . . | $X+2\left(\frac{Z}{2}\right)$, |
| „ Hydrochloric acid . . . | $X+\left(\frac{Y}{2}\right)+\left(\frac{Z}{2}\right)$; |

whence, putting $\alpha'$ as the symbol of the simple weight X, $\varpi'$ as the symbol of the simple weight $\frac{Y}{2}$, and $\chi'$ as the symbol of the simple weight $\frac{Z}{2}$, we arrive at the equation

$$2\alpha'\chi'\varpi'=\alpha'\varpi'^2+\alpha'\chi'^2.$$

This system was pointed out to me, at the time of the appearance of the first part of this memoir, by Professor G. STOKES; it was subsequently noticed by Professor CRUM BROWN ('Philosophical Magazine' for August 1867), and again, more recently, by Professor CLIFFORD in a paper read before the British Association at the Meeting in Belfast, 1874.

This system contains one indeterminate symbol, that is to say, the symbol of one simple weight which cannot be determined from the data given in the equation. If we had before us merely the single equation just considered, we could not possibly say, without an arbitrary (and therefore unmeaning) assumption, to which of the two special systems it was to be referred; and the only rational course to pursue would be to express the result in the indeterminate system, thus keeping both hypotheses before us. But the case is different when we come to consider the expression of the total system of chemical equations, which afford us the means of selection with probability, although not with certainty, between the two determinate systems.

[*] In Part I. Sec. VII. (1) I have termed $\alpha$ the "modulus" of the symbolic system, it being that symbol by which the form of every other symbol of the system is regulated. As I have occasion to use the term "modulus" for another purpose, this use of it would give rise to ambiguity, and I shall substitute for it the term "base.'

unit of hydrogen is not "distributed," and is necessarily (unless we make totally unmeaning assumptions) to be expressed by one prime factor $\alpha$. If, therefore, we are to depart from this principle some ground must be shown for so doing.

The comparative value of two hypotheses may be tested by following these hypotheses out to their consequences, and comparing these consequences with the facts to which the hypotheses are to be applied. If we find that one hypothesis is applicable to all the facts, while the other is applicable to only a portion of those facts, the latter hypothesis is inadmissible. But both hypotheses may include all the facts, and be equally applicable to them, including in each case precisely the same facts. In this case both hypotheses are equally tenable, and we have no means of giving a preference to either. But there is another case: both hypotheses may include all the facts, and so far either may serve our purpose; but one of the two may also do more than we want, covering more ground than is required, and indicating improbable results. In this case that hypothesis is to be preferred the consequences of which most exactly coincide with the facts, namely, the more restricted hypothesis.

Assuming, then, that we are in possession of the symbols of the units of ponderable matter constructed, with logical precision, upon these two several hypotheses, let us consider the consequences to be deduced from the expressions thus assigned to them.

To simplify the problem, let us take the case of the compounds of hydrogen, chlorine, oxygen, carbon.

If $m$ units of any chemical substance be made up of $p$ units of hydrogen, $p'$ units of chlorine, $q$ units of oxygen, and $r$ units of carbon, $m, p, p', q, r$ being positive integers, what are the relations, if any, by which these integers are connected? We shall assume $m, p, p', q, r$ to have no common measure.

I. We will first consider hypothesis $a^2$. On this system we have:—

| | |
|---|---|
| Symbol of Hydrogen . . . . | $a^2$, |
| ,, Chlorine . . . . . | $c^2$, |
| ,, Oxygen . . . . . | $\xi^2$, |
| ,, Carbon . . . . . | $\varkappa^t$, |

the last two symbols being identical in the two systems. The symbol, then, of a unit of matter, expressed by the prime factors $a, c, \xi, \varkappa$, is $a^n c^{n'} \xi^{n''} \varkappa^{n'''}$ [I. Sec. V.], whence, from the conditions of the problem, we have

$$m\, a^n\, c^{n'}\, \xi^{n''}\, \varkappa^{n'''} = pa^2 + p'c^2 + q\xi^2 + r\varkappa^t;$$

whence [I. Sec. V. (10)]

$$nm = 2p,$$

$$n'm = 2p',$$

$$n''m=2q,$$
$$n'''m=tr,$$

and

$$n=\frac{2p}{m}, \text{ a positive integer,}$$
$$n'=\frac{2p'}{m} \quad " \quad "$$
$$n''=\frac{2q}{m} \quad " \quad "$$
$$n'''=\frac{tr}{m} \quad " \quad "$$

It appears from these equations that $m$ is either a factor of $p$, $p'$, and $q$, or is of the form $2m_1$, where $m_1$ is a factor of $p, p'$, and $q$.

Further, since $m$ is a factor of $tr$, if $m$ be a factor of $p, p'$, and $q$, $m$ is prime to $r$, and $m$ is a factor of $t$. If $m=2m_1$, since $m_1$ is prime to $r$, $m_1$ is a factor of $t$. If we assume, therefore, in conformity with the results arrived at as to the symbol of carbon (I. Sec. VII. (2)), $t=1$ and $t=2$ as the most probable assumptions in the case of that symbol, we have in the former case,

$$t=1, n'''=\frac{r}{m}, \text{ and } m=1,$$

in the latter case,

$$t=2, n'''=\frac{2r}{m};$$

whence either $m=1$ or is of the form $m=2m_1$, where $m_1$ is a factor of $r$. But $m$ is prime to $r$; therefore we have on these assumptions only two values for $m$, $m=1$ or $m=2$. It is, however, to be borne in mind that this limitation of the value of $m$ implies a more definite knowledge of the hypothetical density of carbon than we possess. These results are the only consequences to be here deduced from the joint hypotheses that the units of ponderable matter are made up of an integral number of simple weights, and that the unit of hydrogen is constituted of two identical simple weights. The application of these principles will be readily seen in a particular case. For example:—

One unit of trichloracetic acid weighing 7·308 grms. is identical with half a unit of hydrogen, three halves of a unit of chlorine, one unit of oxygen, and some unknown number of units of carbon (to be hypothetically determined) weighing 1·072 grm. Putting the symbol of carbon as $\varkappa^t$, where $t$ is an undetermined integer, we will call this unknown number of units $\frac{\mathrm{R}}{m_1}$. Expressing this identity in an equation, we have

$$m_1 ac^3\xi^2\varkappa^2=\frac{m_1}{2}a^2+\frac{3m_1}{2}c^2+m_1\xi^2+\mathrm{R}\varkappa^t$$

and

$$2m_1\, ac^3\, \xi^2\, \varkappa^2 = m_1\, a^2 + 3m_1\, c^2 + 2m_1\, \xi^2 + 2R\varkappa^t.$$

Comparing this equation with the general equation given above, we perceive that it falls under the case $m=2m_1$, where $m_1$ is a factor of $p$ and $p'$ and $q$.

If we assume $t=1$, which corresponds to the assumption that the weight of a unit (that is, of 1000 cub. centims.) of carbon vapour at 0° and 760 millims. is 0·536, $\frac{R}{m_1}=2$, $R=2$, and $m_1=1$, the equation being

$$2ac^3\xi^2\varkappa^2 = a^2 + 3c^2 + 2\xi^2 + 4\varkappa.$$

If we assume $t=2$, which corresponds to the assumption that the weight of a unit of carbon vapour is 1·072 [I. Sec. VII. Group 2, III.], $\frac{R}{m_1}=1$, $R=1$, $m_1=1$, the equation being

$$2ac^3\, \xi^2\, \varkappa^2 = a^2 + 3c^2 + 2\xi^2 + 2\varkappa^2.$$

Again, one unit of dichloracetic acid is identical with a unit of hydrogen, a unit of chlorine, a unit of oxygen, and (as before) R units of carbon. In this case $m$ is a factor of $p$, $p'$, $q$, the equation being

$$m\, a^2\, c^2\, \xi^2\, \varkappa^2 = m\, a^2 + m\, c^2 + m\, \xi^2 + R\varkappa^t.$$

There are two forms of this equation, corresponding to the hypotheses $t=1$, $t=2$:

$$t=1,\ a^2\, c^2\, \xi^2\, \varkappa^2 = a^2 + c^2 + \xi^2 + 2\varkappa,$$
$$t=2,\ a^2\, c^2\, \xi^2\, \varkappa^2 = a^2 + c^2 + \xi^2 + \varkappa^2.$$

No limitation whatever is imposed by this hypothesis on the value of $n$, $n'$, $n''$, $n'''$. Two cases may be discriminated:

$$(1)\ m=1,\ n=2p,\ n'=2p',\ n''=2q,\ n'''=r,$$
$$(2)\ m=2,\ n=p,\quad n'=p',\quad n''=q,\quad n'''=r;$$

it being always remembered that $m, p, p', q, r$ have no common measure.

It hence appears that every combination of the letters $a, c, \xi, \varkappa$ of the form $a^n c^{n'} \xi^{n''} \varkappa^{n'''}$ is the symbol of a unit of ponderable matter capable of being resolved into some number of units of hydrogen, chlorine, oxygen, and carbon, on the assumption here made as to the constitution of those elements.

Now the symbols of all known compounds of these elements are to be found among these combinations, with their density and composition correctly indicated to us. Hence the hypothesis that the symbol of hydrogen is $a^2$ covers the whole ground, and is sufficient for our purpose. If it be defective, it is not by reason of its insufficiency. But there are other points to be considered.

II. On the hypothesis $\alpha$, namely, that the symbol of hydrogen is to be expressed by one prime factor, we have :—

| | |
|---|---|
| Symbol of Hydrogen . . . . . . | $\alpha$, |
| ,, Chlorine . . . . . . | $\alpha\chi^2$, |
| ,, Oxygen . . . . . . . | $\xi^2$, |
| ,, Carbon . . . . . . . | $\varkappa^t$, |

the symbol of a unit of matter as expressed by the prime factors $\alpha$, $\chi$, $\xi$, $\varkappa$ being $\alpha^n\chi^{n'}\xi^{n''}\varkappa^{n'''}$.

From the conditions of the problem we have

$$m\alpha^n\chi^{n'}\xi^{n''}\varkappa^{n'''}=p\alpha+p'\alpha\chi^2+q\xi^2+r\varkappa^t;$$

whence

$$\begin{aligned} nm&=p+p',\\ n'm&=2p',\\ n''m&=2q,\\ n'''m&=tr, \end{aligned}$$

and

$$\begin{aligned} n&=\frac{p+p'}{m}, \text{ a positive integer,}\\ n'&=\frac{2p'}{m} \quad ,, \quad ,,\\ n''&=\frac{2q}{m} \quad ,, \quad ,,\\ n'''&=\frac{tr}{m} \quad ,, \quad ,, \end{aligned}$$

There are two cases.

I. $m$ is a factor of $p$, $p'$, $q$ and is prime to $r$.

If $t=1$, $n'''=\frac{r}{m}$, and $m=1$, since $p$, $p'$, $q$, $r$ are so taken as to have no common measure, and

$$\begin{aligned} n&=p+p',\\ n'&=2p',\\ n''&=2q,\\ n'''&=r. \end{aligned}$$

If $t=2$, $n'''=\frac{2r}{m}$. In this case $m$ may have one of two values, $m=1$ and $m=2$. In the case $m=1$, we have

$$\begin{aligned} n&=p+p',\\ n'&=2p',\\ n''&=2q,\\ n'''&=2r. \end{aligned}$$

In the case $m=2$,

$$n=\frac{p+p'}{2},$$
$$n'=p',$$
$$n''=q,$$
$$n'''=r.$$

II. $m$ is of the form $2m_1$, where $m_1$ is a factor of $p'$ and $q$. In this case we have

$$n=\frac{p+p'}{2m_1},$$
$$n'=\frac{p'}{m_1},$$
$$n''=\frac{q}{m_1},$$
$$n'''=\frac{tr}{2m_1}.$$

(1) If $t=1$, $n'''=\frac{r}{2m_1}$, $m_1=1$, and

$$n=\frac{p+p'}{2},$$
$$n'=p',$$
$$n''=q,$$
$$n'''=\frac{r}{2}.$$

(2) If $t=2$, $n'''=\frac{r}{m_1}$, $m_1=1$, and

$$n=\frac{p+p'}{2},$$
$$n'=p',$$
$$n''=q,$$
$$n'''=r.$$

It appears therefore that, in the case of the hypothesis that the symbol of hydrogen is expressed by one prime factor $\alpha$, $m$ (as in the case of the previous hypothesis) may have two values and two values only, $m=1$, $m=2$. If $m=1$ no restriction is placed on the values of the integers $p$, $p'$, $q$, $r$; but if $m=2$, $p+p'$ must be an even number.

Referring again to the system of equations

$$nm=p+p',$$
$$n'm=2p',$$
$$n''m=2q,$$
$$n'''m=tr,$$

we have from these equations

$$\frac{n'}{n}=\frac{2p'}{p+p'}.$$

Now the values of the fraction $\frac{2p'}{p+p'}$ for all positive and integral values of $p$ and $p'$ (whatever be the value of $m$) are comprehended between the limits 0 and 2.

If $p'=0$,

$$\frac{2p'}{p+p'}=0.$$

If $p=0$,

$$\frac{2p'}{p+p'}=2.$$

It hence appears that if the symbol $\alpha^n\chi^{n'}\xi^{n''}\varkappa^{n'''}$ is to satisfy the conditions given in the equation

$$m\alpha^n\chi^{n'}\xi^{n''}\varkappa^{n'''}=p\alpha+p'\alpha\chi^2+q\xi^2+r\varkappa^t,$$

a restriction is placed by the conditions of the problem upon the values of the integers $n$ and $n'$, which must be so selected that $\frac{n'}{n}$ shall not be $<0$ or $>2$.

If $\frac{n'}{n}=0$, $p'=0$, and the equation becomes

$$m\alpha^n\xi^{n''}\varkappa^{n'''}=p\alpha+q\xi^2+r\varkappa^t.$$

If $\frac{n'}{n}=2$, $p=0$, and the equation becomes

$$m\alpha^n\chi^{2n}\xi^{n''}\varkappa^{n'''}=p'\alpha\chi^2+q\xi^2+r\varkappa^t.$$

No restriction exists on the values of $n''$ and $n'''$. We may notice the two forms :—

$$(1)\qquad \begin{aligned} m&=1,\\ n&=p+p',\\ n'&=2p',\\ n''&=2q,\\ &\begin{cases} n'''=r,\\ n'''=2r.\end{cases}\end{aligned}$$

$$(2)\qquad \begin{aligned} m&=2,\\ n&=\frac{p+p'}{2},\\ n'&=p',\\ n''&=q,\\ &\begin{cases} n'''=r,\\ n'''=\frac{r}{2}.\end{cases}\end{aligned}$$

For the interpretation of the symbol of a unit of matter it is essential that we should be informed of the conditions which that unit is to satisfy, apart from which no interpretation is possible, for the same symbol may have its origin in many ways. The way in which the symbol originates is defined by the equations; thus, in the present case, to interpret the symbols $\alpha^n\chi^{n'}\xi^{n''}\kappa^{n'''}$,

$n$ is the ratio of the sum of the units of hydrogen and chlorine formed to the number of units of matter decomposed;

$n'$ is twice the ratio of the number of units of chlorine formed to the number of units of matter decomposed;

$n''$ is twice the ratio of the number of units of oxygen formed to the number of units of matter decomposed;

$n'''$ is $t$ times the ratio of the number of units of carbon formed to the number of units of matter decomposed, $t$ being (probably) 1 or 2.

For example, to interpret the symbols $\alpha\chi$, $\alpha\xi$. In both cases $n=1$. That is to say, $\alpha\chi$ and $\alpha\xi$ are symbols of two units of matter such that the sum of the units of hydrogen and chlorine formed by their respective decompositions is equal to the number of units thus decomposed. In the case of $\alpha\chi$, $n'=1$, $n''=0$, $n'''=0$, that is to say, the ratio of the units of chlorine formed to the units of matter decomposed is as $1:2$. No oxygen and no carbon are formed in the decomposition. In the case of $\alpha\xi$, $n'=0$, $n''=1$, $n'''=0$. The ratio of the units of oxygen formed to the units of matter decomposed is as $1:2$, no chlorine and no carbon are formed. These properties characterize the respective units of hydrochloric acid and of water. Again, in the case of the symbol $\alpha^2\chi\xi\kappa$ (the symbol of the chloride of acetyl), $n=2$. The sum of the units of hydrogen and chlorine formed by the decomposition of the substance is twice the number of units of the substance decomposed. $n'=1$, the ratio of the units of chlorine found to the units of matter decomposed is as $1:2$. $n''=1$, the ratio of the units of oxygen found to the units of matter decomposed is also $1:2$. $n'''=1$, the ratio of the units of carbon formed to the units of matter decomposed is as $1:t$, where $t$ is assumed to be 1 or 2. We hence are always able, from inspection of the symbol, to reconstitute the equation whence that symbol was derived.

The preceding reasoning may be extended to the general case. Let $m$ units of any chemical substance $\alpha^n\chi^{n'}\omega^{n'_1}\nu^{n'_2}\ldots\xi^{n''}\theta^{n''_1}\lambda^{n''_2}\ldots\kappa^{n'''}\delta^{n'''_1}\ldots$ be identical with $p$ units of hydrogen $\alpha$, $p'$ units of chlorine $\alpha\chi^2$, $p''$ units of iodine $\alpha\omega^2$, $p'''$ units of nitrogen $\alpha\nu^2\ldots$, $q$ units of oxygen $\xi^2$, $q'$ units of sulphur $\theta^2$, $q''$ units of selenium $\lambda^2\ldots$, $r$ units of carbon $\kappa$, $r'$ units of mercury $\delta$, $r''$ units of zinc $\zeta\ldots$, so that

$$m\alpha^n\chi^{n'}\omega^{n'_1}\nu^{n'_2}\ldots\xi^{n''}\theta^{n''_1}\lambda^{n''_2}\ldots\kappa^{n'''}\delta^{n'''_1}\ldots=p\alpha+p'\alpha\chi^2+p''\alpha\omega^2+p'''a\nu^2+\ldots$$
$$+q\xi^2+q'\theta^2+q''\lambda^2+\ldots+r\kappa+r'\delta+r''\zeta+\ldots,$$

we have then the following equations connecting the positive integers $m, n, n', \ldots p, p', p'', \ldots$

$$mn=p+p'+p''+p'''+\dots$$

$$mn'=2p', \quad n'=\frac{2p'}{m},$$

$$mn'_1=2p'', \quad n'_1=\frac{2p''}{m},$$

$$mn'_2=2p''', \quad n'_2=\frac{2p'''}{m},$$

. . . . . . .    . . . .

. . . . . . .    . . . .

$$mn''=2q, \quad n''=\frac{2q}{m},$$

$$mn''_1=2q', \quad n''_1=\frac{2q'}{m},$$

$$mn''_2=2q'', \quad n''_2=\frac{2q''}{m},$$

. . . . . . .    . . . .

. . . . . . .    . . . .

$$mn'''=r, \quad n'''=\frac{r}{m},$$

$$mn'''_1=r', \quad n'''_1=\frac{r'}{m},$$

$$mn'''_2=r'', \quad n'''_2=\frac{r''}{m},$$

. . . . . . .    . . . .

. . . . . . .    . . . .;

whence, putting

$$\begin{aligned}
\Sigma(p') &= p'+p''+p'''+\dots,\\
\Sigma(q) &= q+q'+q''+\dots,\\
\Sigma(r) &= r+r'+r''+\dots,\\
\Sigma(n') &= n'+n'_1+n'_2+\dots,\\
\Sigma(n'') &= n''+n''_1+n''_2+\dots,\\
\Sigma(n''') &= n'''+n'''_1+n'''_2+\dots,
\end{aligned}$$

we have

$$n=\frac{p+\Sigma(p')}{m},$$

$$\Sigma(n')=\frac{2\Sigma(p')}{m},$$

$$\Sigma(n'')=\frac{2\Sigma(q)}{m},$$

$$\Sigma(n''')=\frac{\Sigma(r)}{m}.$$

And reasoning as before, since $m$ is prime to some one, at least, among the numbers $p, p', p'', p''', \ldots q, q', q'', q''' \ldots r, r', r'' \ldots$, if $m$ be prime to some one among the numbers $r, r', r'', \ldots m=1$. If $m$ be prime to any one among the numbers $p', p'', p''', \ldots q, q', q''$, and be not prime to any one among the numbers $r, r', r'' \ldots$, $m=2$. If $m$ be not prime to any one among the numbers $p', p'', p''', \ldots q, q', q'', \ldots r, r', r'', \ldots m$ is also not prime to $p$, which is contrary to the construction. Therefore $m=1$ or $m=2$.

$$\text{If } m=1, \quad n=p+\Sigma(p'),$$

$$\Sigma(n')=2\Sigma(p');$$

$$\text{if } m=2, \quad n=\frac{p+\Sigma(p')}{2},$$

$$\Sigma(n')=\Sigma(p'),$$

the condition being that $p+\Sigma(p')$ is to be an even number.

We have also

$$\frac{\Sigma(n')}{n}=\frac{2\Sigma(p')}{p+\Sigma(p')};$$

whence, in addition to the conditions

$$\frac{n'}{n} \text{ not } <0 \text{ not } >2,$$

$$\frac{n'_1}{n} \quad ,, \quad ,, \quad ,,$$

$$\frac{n'_2}{n} \quad ,, \quad ,, \quad ,,$$

we have

$$\frac{\Sigma(n')}{n} \text{ not } >2.$$

The symbol $\alpha^{n}\chi^{n'}\omega^{n'_1}\ldots\xi^{n''}\theta^{n''_1}\ldots\varkappa^{n'''}\delta^{n'''_1}\ldots$ is to be interpreted on the same principles as the symbol $\alpha^{n}\chi^{n'}\xi^{n''}\varkappa^{n'''}$ previously considered. Thus to interpret the symbol $\alpha\chi\omega$, here $n=1$, the sum of the units of hydrogen, chlorine, and iodine formed by the decomposition of the substance when decomposed into these elements is equal to the number of units of the substance thus decomposed, $n'=1$, $n'_1=1$; of the units of hydrogen, chlorine, iodine thus formed $\frac{1}{2}$ are units of iodine and $\frac{1}{2}$ units of chlorine, no hydrogen being formed. These are the characteristic properties of the unit of chloride of iodine.

This reasoning may be further extended so as to include the case of the elements phosphorus $\alpha^2\phi^4$ and other elements similar to it in form.

Both hypotheses, therefore, so far agree that each covers the whole ground, and includes all known compounds of carbon, hydrogen, oxygen, chlorine ... But the system of combinations constructed upon hypothesis $\alpha$ does not include those pretended compounds of carbon, hydrogen, oxygen, chlorine (recognized as possible compounds of these elements in the other system) in the case of which the sum of the units of hydrogen, chlorine... (and generally the sum of the units of the dyad elements) formed by the decomposition of two units of the compound is other than an even number. Hypothesis $\alpha$ affords no means of manufacturing these compounds; so that we are led to the conclusion that such things are to be regarded as mere "non-entities" or "monstrosities," combinations of incompatible ingredients, which cannot be made up, by any known chemical operations, of the matter of those actual chemical elements out of which all known chemical existences are constructed*. It is hardly necessary to observe that such things cannot, as a matter of fact, be made—the limitation introduced by hypothesis $\alpha$ being the celebrated law of Even Numbers, discovered as an empirical truth by LAURENT and GERHARDT, which is here referred to its origin, and deductively established, as a consequence of this hypothesis.

Were we to proceed to enumerate the combinations of hydrogen, chlorine, nitrogen, and the other dyad elements, as indicated by the two hypotheses respectively, and compare the systems thus indicated with the systems actually existing, we should find that the combinations indicated in the case of hypothesis $\alpha^2$ were twice as numerous as in the case of hypothesis $\alpha$; and we should, in fact, on this hypothesis ($\alpha^2$) make a blunder in every alternate statement, knowing neither more nor less about the result than we do in tossing a halfpenny whether it will fall head or tail—a position very different to that of one who, having obtained information that the coin is weighted, never makes any such mistakes at all, which is our position on hypothesis $\alpha$. That the conclusions to which we are thus brought as to the nature and relations of these dyad elements are truly of a remarkable character is not to be denied; but it is a sufficient reply to those persons who make difficulties on this score, that they may just as reasonably object to the facts on which these conclusions are based, which are equally remarkable with the conclusions; and, moreover, that the value of an hypothesis is to be tested, not by the coincidence of its conclusions with our previous expectations, but by their agreement with the facts which we wish to explain.

Now it may be contended, admitting it to be true that hypothesis $\alpha$ puts a correct limitation upon these combinations, yet this advantage is gained at too great a cost. It rejects indeed (it may be said), and rightly, the combinations rejected by the empirical law of Even Numbers, but, in so doing, opens the door of the system to a large number of compounds totally unknown to us. Thus while we eradicate one set of impossible existences, we introduce another. Where, it is asked, are the elements $\chi$, $\omega$, $\nu$..., or

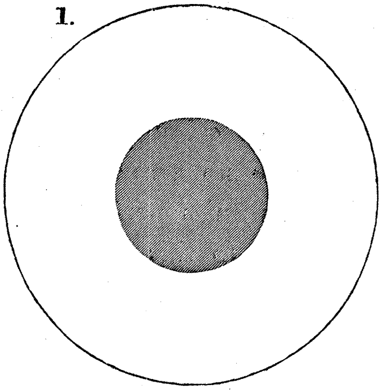
---

* "Desinit in piscem mulier formosa superne." The construction of the unit of a chemical substance to be resolved into two units of hydrogen and three units of chlorine presents the same order of difficulties as the combination of the head of a woman with the tail of a fish. We cannot imagine how such a thing could be produced!

such combinations of these elements as the units $\alpha\chi^3$, $\chi\xi$, $\chi^\nu$, $\varkappa\chi^2 \ldots$ and the like? But this argument does not come to much. Our only choice lies between the two hypotheses in question, and hypothesis $a^2$ is attended with far graver difficulties; for in this system the real and unreal combinations are mixed up together, which is not the case on hypothesis $\alpha$. On this latter assumption we do not introduce among the units made up of the matter of our actual elemental bodies the unit of any chemical substance whatever which cannot be thus made up; so that the system with which we have to work makes no false assertion, but is from the beginning properly constructed. A sharp line of demarcation is drawn in it between what is and what is not, our actual system of chemical combinations appearing to us as a fragment of a wider and unrealized system, of which it is a part and in which it is comprehended.

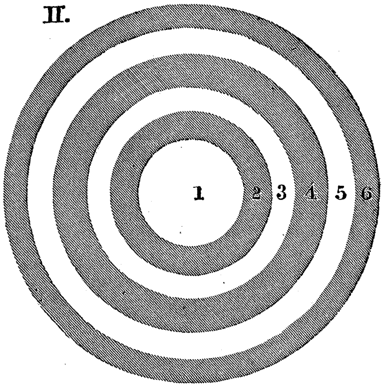
The relation of the two hypotheses will be readily appreciated from the following diagrams:—

I.



Hypothesis $\alpha$.

II.



Hypothesis $a^2$.

Figure I. represents to us the system of combinations on hypothesis $\alpha$. The total area included within the outer contour indicates what I may term the region of possible combinations—that is to say, of the combinations, actual or conceivable, of the simple weights of the system $\alpha, \chi, \xi, \omega, \nu \ldots$ by which we may consider it to be occupied. This region is divided into two subordinate regions—the region of actual existences, namely, the region occupied by the combinations made up of the matter of our actual elemental bodies, hydrogen, chlorine, iodine, nitrogen, oxygen, carbon, and the like, indicated by the shaded space; and an unknown or unexplored region, external to this, containing the combinations of these same simple weights which are not thus made up and of which we have no actual representatives. Were we to write down in each region the symbols of the combinations by which we thus consider it to be inhabited, we should place within the shaded space the symbols satisfying the conditions given by the law of even numbers, and in the open space the symbols which do not satisfy those conditions.

Figure II. is constructed on the same principles, to indicate the combinations on hypothesis $a^2$. The area within the outer line comprehends the region of possible combinations, actual or conceivable, of the simple weights of the system* $a, c, i, n, \xi, \ldots$, all of which are constituted of the matter of our actual elemental bodies, and are capable of being resolved into them. But this region is divided into a system of annular spaces, of which only each alternate ring is actually inhabited. The occupied regions are indicated by the shaded spaces, and are assumed to be tenanted by those substances which satisfy the law of even numbers; while the unexplored regions, indicated by the open rings, are occupied by the imaginary substances which do not satisfy this law. It is therefore a mere mistake to consider hypothesis $\alpha$ to be in any sense more hypothetical than hypothesis $a^2$. Both hypotheses indicate to us the possible existence of a system of unreal things; but there is a wide difference in the relation subsisting between the realities and unrealities on the two views respectively. If we consider the operations of nature to be directed, as a flight of arrows at a target, to the construction out of the matter of our elemental bodies of a system of chemical substances made up of the matter of those elements, on hypothesis $\alpha$ the arrows go direct to the mark, this aim is properly attained; if no arrows appear in the outer rings of the target, it is that they lie wide of the mark; whereas on hypothesis $a^2$, while the object aimed at is the same, the arrows are placed exclusively in the alternate or coloured rings of the target, 2, 4, 6, . . ., while the white rings, 1, 3, 5, . . ., have no arrows placed in them at all†. The former hypothesis, therefore, leads to results perfectly consistent with the view that the operations of nature are directed to this end, while the results to which we are brought by the latter hypothesis are absolutely inconsistent with this view.

Although the actual existence of those substances which lie external to our system is by no means a necessary consequence of our hypothesis (for we certainly cannot expect to be able to do all that is possible to be done), and the construction of these substances may be a problem altogether beyond the range of our experimental powers, yet it must be admitted by any who hold to the principles of this method that the appearance upon the scene of these "ideal existences" is at least a possible phenomenon; and it is not without interest to consider our position in regard to this contingency. Two reasons

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* *i. e.* $\frac{\mathrm{H}}{2}, \frac{\mathrm{Cl}}{2}, \frac{\mathrm{I}}{2}, \frac{\mathrm{N}}{2}, \frac{\mathrm{O}}{2}, \ldots$

† In figure I. we should have within the central shaded space in the case of the combinations $\alpha^n \chi^{n_1} \omega^{n_2} \ldots$, those combinations in which the ratios $\frac{n_1}{n}, \frac{n_2}{n}, \ldots$ were severally not $<0$ not $>2$, while in the open space the combinations would appear in which this condition was not satisfied. In figure II., considering the symbol $a^m c^{m_1} i^{m_2}, \ldots$, we should have in the shaded spaces 2, 4, 6, . . ., the combinations of these letters taken 2 and 2, 4 and 4, 6 and 6, . . . together, which combinations are represented in our actual system of things; while in the open spaces, 1, 3, 5, . . . we should have the combinations of these same simple weights taken singly, 3 and 3, 5 and 5, . . ., which are non-existent combinations.

may be given for believing this to be possible: the one, of an abstract kind, derived from the fundamental property of chemical symbols given in the equation

$$\alpha\chi^2=\alpha+2\chi,$$

whence it may be argued that, as we can theoretically resolve $\alpha\chi^2$ into $\alpha$ and $\chi$ and $\chi$, we are not justified in shutting our eyes to the possibility that the unit of chlorine may actually be resolved into a unit of hydrogen and two units of the (as yet) unknown element $\chi$. The argument is just, but the presumption thus raised is excessively slight; for we are able to make (on any hypothesis) so very small a proportion of the vast number of chemical substances, the possible existence of which is similarly indicated to us, that we have no expectation, capable of being estimated, of making any one substance in particular. In short, the thing is possible; but on these grounds alone we cannot expect to be able to do it. The second argument is of far greater weight. It may be put thus:—All persons would admit the validity of our conclusions if, having first detected the element $\chi$, thus latent in its compounds, we were able to verify our hypothesis by digging it out. $\chi$ having been discovered, there would be but little doubt as to the existence of $\omega$ and $\nu$. We have not got quite to this point, but nevertheless we are not so very far from it; for we can show clearly that the application of the very same formal principles of reasoning to existing facts would have enabled us similarly to detect, prior to their isolation, the latent existence in their compounds of elements which have actually been isolated*.

Were we able to conduct our chemical experiments only between certain fixed limits of temperature (let us say between 0° and 300° C.), a barrier would be placed upon our researches. We might still conduct innumerable experiments, but the science which resulted from them would be essentially different to our present chemistry. Between these limits of temperature the appearance of the element carbon in any system of chemical transformations is a rare phenomenon indeed; and it is by no means difficult to place ourselves, by an effort of imagination, in the position of chemists who should have a very varied and extensive knowledge of the chemical properties of the compounds of carbon, and yet be totally unaware of the existence of that element. Let us imagine, then, some unknown world, some Laputa (devoted to philosophy) in which the chemistry

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* Chemists are placed in regard to the ideal elements $\chi$, $\omega$, $\beta$, $\nu$, and those combinations of them which cannot be resolved into our actual elemental bodies, in a very similar position to that occupied by astronomers (if I may venture on such a comparison) towards the planet Neptune prior to its detection by the telescope. The existence of the planet Neptune could be inferred, as a highly probable contingency, from the perturbations of the orbit of Uranus; and yet this planet might have remained for ever unseen by man. So, too, the existence of the element $\chi$ may be similarly inferred from the peculiar forms of chemical metamorphosis; and yet this element may never be isolated as an independent reality. It may exist and exercise (so to say) an appreciable influence upon the movements of our system, and nevertheless lie far beyond the range of facts accessible to our methods of experiment. No scientific hypothesis can have any claim whatever upon our notice which is incapable of experimental verification (a truth too often forgotten in chemical speculations); but this verification may be a very tardy process, and involves numerous trials and numerous failures before in either sense (so as to enable us to deny or affirm the hypothesis) it is accomplished.

was of this order. How would the Laputians proceed with the scientific construction of the symbols of the units of matter? Two units of marsh-gas (they would say) are identical with three units of hydrogen and a unit of acetylene. Having solved a similar problem in the case of ammonia [I. Sec. VII. (8)], they have no difficulty in at once writing down as follows the symbols of these units on the two hypotheses $\alpha$ and $a^2$ respectively:—

| | Hypothesis $\alpha$. | Hypothesis $a^2$. | Weight in grm. |
|---|---|---|---|
| Hydrogen . . . . . . . | $\alpha$ | $a^2$ | 0·089 |
| Marsh-gas . . . . . . . | $\alpha^2\varkappa$ | $a^3\upsilon$ | 0·704 |
| Acetylene . . . . . . . | $\alpha\varkappa^2$ | $\upsilon^2$ | 1·161 |

The simple weight $\upsilon$, which appears in the symbols as expressed on hypothesis $a^2$, weighs 0·58 grm.

We shall suppose our chemists to be in possession of those substances of which the matter is identical with the matter of acetylene, hydrogen, oxygen, chlorine, and our other elements, the material of an ample chemistry. Proceeding with the construction of the symbols of the units of matter on the two hypotheses, they would find both to be applicable as indicated in the following examples:—

| | Hypothesis $\alpha$. | Hypothesis $a^2$. |
|---|---|---|
| Benzol . . . . . . . . . | $\alpha^3\varkappa^6$ | $\upsilon^6$ |
| Olefiant gas . . . . . . . . | $\alpha^2\varkappa^2$ | $\alpha^2\upsilon^2$ |
| Allylene . . . . . . . . . | $\alpha^2\varkappa^3$ | $\alpha\upsilon^3$ |
| Methyl . . . . . . . . . | $\alpha^3\varkappa^2$ | $\alpha^4\upsilon^2$ |
| Propylene . . . . . . . . | $\alpha^3\varkappa^3$ | $\alpha^3\upsilon^3$ |
| Ethyl . . . . . . . . . . | $\alpha^5\varkappa^4$ | $\alpha^6\upsilon^4$ |
| Allyl . . . . . . . . . . | $\alpha^5\varkappa^6$ | $\alpha^4\upsilon^6$ |
| Formic acid . . . . . . . . | $\alpha\varkappa\xi^2$ | $\alpha\upsilon\xi^2$ |
| Methylic alcohol . . . . . | $\alpha^2\varkappa\xi$ | $\alpha^3\upsilon\xi$ |
| Oxide of ethylene . . . . | $\alpha^2\varkappa^2\xi$ | $\alpha^2\upsilon^2\xi$ |
| Alcohol . . . . . . . . . | $\alpha^3\varkappa^2\xi$ | $\alpha^4\upsilon^2\xi$ |
| Benzylic alcohol . . . . . | $\alpha^4\varkappa^7\xi$ | $\alpha\upsilon^7\xi$ |
| Acetic acid . . . . . . . . | $\alpha^2\varkappa^2\xi^2$ | $\alpha^2\upsilon^2\xi^2$ |
| Glycol . . . . . . . . . | $\alpha^3\varkappa^2\xi^2$ | $\alpha^4\upsilon^2\xi^2$ |
| Glycerine . . . . . . . . | $\alpha^4\varkappa^3\xi^2$ | $\alpha^5\upsilon^3\xi^2$ |
| Anhydrous acetic acid . . . | $\alpha^3\varkappa^4\xi^3$ | $\alpha^2\upsilon^4\xi^3$ |
| Acetic peroxide . . . . . | $\alpha^3\varkappa^4\xi^4$ | $\alpha^2\upsilon^4\xi^4$ |
| Lactic acid . . . . . . . . | $\alpha^3\varkappa^3\xi^3$ | $\alpha^3\upsilon^3\xi^3$ |
| Chloride of methyl . . . . | $\alpha^2\varkappa\chi$ | $\alpha^2\upsilon c$ |
| ,, ,, ,, . . . . | $\alpha^2\varkappa\chi^2$ | $\alpha\upsilon c^2$ |
| Chloroform . . . . . . . | $\alpha^2\varkappa\chi^3$ | $\upsilon c^3$ |

| | | |
|---|---|---|
| Chloride of ethylene | $\alpha^3\varkappa^2\chi^2$ | $\alpha\upsilon^2c^2$ |
| Chloride of acetyl | $\alpha^2\chi\varkappa^2\xi$ | $\alpha\upsilon^2c\xi$ |
| Chloracetic acid | $\alpha^2\chi\varkappa^2\xi^2$ | $\alpha\upsilon^2c\xi^2$ |
| Chloriodide of ethylene | $\alpha^3\chi\omega\varkappa^2$ | $\alpha^2\upsilon^2ci$ |
| Hydrocyanic acid | $\alpha\nu\varkappa$ | $\upsilon n$ |
| Methylamine | $\alpha^3\nu\varkappa$ | $\alpha^3\upsilon n$ |
| Trimethylamine | $\alpha^5\nu\varkappa^3$ | $\alpha^3\upsilon^3n$ |

It is thus evident that the Laputian philosophers might have proceeded a long way indeed in the study of the metamorphoses of the compounds of carbon, and yet take a very different view of the nature of these compounds to that taken by ourselves. In their eyes the group of hydrocarbons would appear as compounds of hydrogen and the element acetylene, of which they would possess in benzol an allotropic modification. Besides these they might have innumerable compounds of acetylene and their other elements, of the kind indicated in the Table.

Proceeding with their deductive investigations, they would arrive at the law of even numbers, and applying this critical test to the compounds of acetylene, would arrive at the following equations:—

$$\begin{aligned}
2\alpha^2\varkappa &= 3\alpha+\alpha\varkappa^2,\\
2\alpha^2\varkappa^3 &= \alpha+3\alpha\varkappa^2,\\
2\alpha^3\varkappa^3 &= 3\alpha+3\alpha\varkappa^2,\\
\alpha^3\varkappa^4 &= \alpha+2\alpha\varkappa^2,\\
2\alpha^2\varkappa\chi^2 &= 2\alpha+\alpha\varkappa^2+\alpha\chi^2,\\
2\alpha^2\varkappa\chi^3 &= 3\alpha\chi^2+\alpha\varkappa^2,\\
2\alpha^2\varkappa\chi^2\xi &= \alpha+2\alpha\chi^2+\alpha\varkappa^2+\xi^2,\\
2\alpha^2\chi\varkappa^2\xi &= \alpha+\alpha\chi^2+2\alpha\varkappa^2+2\xi^2,\\
2\alpha^3\chi\omega\varkappa^2 &= 2\alpha+\alpha\chi^2+\alpha\omega^2+2\alpha\varkappa^2,\\
&\cdots\cdots\cdots\cdots\\
&\cdots\cdots\cdots\cdots
\end{aligned}$$

The Laputian philosophers may be assumed to have had great reliance on their deductive processes, and would doubtless have said at once that these facts afforded a solid ground of preference in favour of their hypothesis $\alpha$—indeed that the other hypothesis was relatively untenable. Nevertheless we may believe them to have felt some satisfaction when, on descending from their floating island to our earth, we showed them the process by which acetylene was manufactured by BERTHELOT from hydrogen and carbon, and thus turned their hypothesis into a demonstrated truth.

There is, however, a difficulty in the way of the unqualified acceptance of hypothesis $\alpha$, of a very real character, namely that it does not, *primâ facie*, appear to be in perfect concordance with facts. Now the facts to which this remark applies are of two classes:—(1) The assumed densities of gases which are not in accordance with either of the two hypotheses $\alpha$ and $a^2$, and (2) assumed densities which are not *primâ facie* in accordance with hypothesis $\alpha$, but which are not discordant with hypothesis $\alpha^2$. In the former case the units of matter cannot be expressed by an integral number of prime factors, either on hypothesis $\alpha$ or on hypothesis $\alpha^2$; in the second case the units of matter can be expressed by an integral number of prime factors on hypothesis $\alpha^2$, but cannot be so expressed on hypothesis $\alpha$. The question has been (up to a certain point) discussed in Part I. Section VIII., where a Table is given of these exceptions [Part I. Section VIII. (3)].

I shall not enter on the former class of exceptions, not only for the reason that the difficulties presented by them are not peculiar to this method, but also that these obstacles are rapidly being removed by a careful scrutiny of the facts, and so many such obstacles have actually disappeared that the removal of the rest must be regarded simply as a question of time and trouble. The second class of exceptions, however, stands on a very different footing. They go to the very root of the matter, and no rational explanation of these exceptions has (so far as I am aware) been ever suggested from the present point of view of chemistry. We will fix our attention on two salient examples, the binoxide and tetroxide of nitrogen.

Assuming the gaseous densities of these substances as given by various observers to be the true densities of homogeneous gases, it must be admitted that the ponderable matter of two units of binoxide of nitrogen is identical with the ponderable matter of a unit of nitrogen and a unit of oxygen; and also it must be admitted that the ponderable matter of two units of tetroxide of nitrogen is identical with the ponderable matter of a unit of nitrogen and two units of oxygen. The facts here stated are incompatible with the expression of the symbols of the units of the binoxide of nitrogen and of the tetroxide of nitrogen respectively by an integral number of prime factors, on the hypothesis that the symbol of hydrogen is expressed as $\alpha$ [Part I. Section VIII. (2)]; for we cannot find, on that hypothesis, any positive and integral solutions of the indeterminate equations connecting the integers, which are the indices of the prime factors by which these symbols are expressed.

Now if this be regarded as a true statement of the facts, it must be allowed that the binoxide and tetroxide of nitrogen are not only (what indeed, perhaps without exception, they are) the most curious of all chemical substances*, but are absolutely unique objects not made in the same way as other things, or turned out of the same workshop with them; in short (as I before said) they must be "chemical monstrosities," "lusus

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* LAURENT endeavours to get out of the difficulty by drawing a distinction between the molecule and (what I have termed) the unit. The unit, he says, is indeed NO, but the molecule is $N_2 O_2$. As, however, a molecule can only be defined by referring it to the unit and saying that in all cases two units of matter are constituted of an equal number of molecules, this explanation is not very satisfactory, although it is a path in which LAURENT has been contentedly followed by the greater number of chemists. LAURENT was fully alive to the difficulties of the case, as appears from the following sentence :—"Unfortunately nitric oxide and peroxide of nitrogen are such singular bodies that it appears somewhat difficult to discover their analogues," taken together with the note attached ('Chemical Method,' translation, Cavendish Society, 1855, p. 82).

naturæ," which cannot be generated by any known process of generation. Without venturing to impose any limit on the powers of nature, we yet should think twice about the evidence before committing ourselves to belief in a mermaid; and I must confess that I feel a similar hesitation in believing in the existence of any chemical substance made out of hydrogen, oxygen, and nitrogen which cannot be made by the operations $\alpha$, $\xi$, and $\nu$. We are not, however, driven into this corner.

First, we may notice in regard to one of these two substances, namely, hyponitric acid, that the observed density of the gas is not a constant, but varies with the temperature at which the observation is made. This point has been observed in three independent sets of experiments by PLAYFAIR and WANKLYN, by MÜLLER and by DEVILLE, the results being in the main concordant. When the density is taken at a low temperature, it approximates to, although it does not reach, the number corresponding to the symbol $\alpha\nu^2\xi^4$; thus we have:—

| Observer. | Temperature. | Density (Air 1). | Density (Hyd. 1). |
|---|---|---|---|
| DEVILLE . . . . . . . . | 26°·7 | 2·65 | 38·24 |
| MÜLLER . . . . . . . . | 20 | 2·70 | 38·96 |
| PLAYFAIR and WANKLYN . . | 24·5 | 2·52 | 36·39 |
| " " . . | 11·3 | 2·64 | 38·19 |
| " " . . | 4·2 | 2·59 | |

The density corresponding to the symbol $\alpha\nu^2\xi^4$ is 46·0, showing a difference between this number and the lowest observed density of about 15 per cent.

As the temperature of the gas increased the density diminished, until the observed density (in the observation made) attained a numerical value of 1·57, after which the observations ceased.

In the following Table these densities are calculated on the hydrogen scale, and also in the usual manner, the density of air being assumed as 1:—

| Temperature. | Density (Air 1). | Density (Hydrogen 1). |
|---|---|---|
| 100°·1 | 1·68 | 24·24 |
| 111·3 | 1·65 | 23·80 |
| 121·5 | 1·62 | 23·35 |
| 135·0 | 1·60 | 23·14 |
| 154·0 | 1·58 | 22·80 |
| 183·2 | 1·57 | 22·60 |

Now the theoretical density of the tetroxide of nitrogen symbolized as $\alpha\nu^2\xi^4$ is on the hydrogen-scale 46, and on the air-scale 3·19. Half this density, therefore, is on the hydrogen-scale 23, and on the air-scale 1·59. The conclusion to be drawn from these numbers, therefore, is that the gas undergoes a gradual expansion with the increase of temperature until at about 150° this limit is attained, so that the constitution of the gas at the lower temperature is essentially different from its constitution at the higher temperature. But in what does this difference consist?

If we are to discuss such questions with advantage, we must be content to advance from the known to the unknown, and base our reasoning upon analogous cases with which we are already acquainted. Here, again, I shall venture to give an illustration of the subject from the chemistry of Laputa.

The Laputians, like ourselves, had their chemical difficulties. Two problems especially had perplexed the heads of their philosophers (although, indeed, some of their chemists saw no difficulty in them at all). They had in their possession two chemical substances, to the units of which they had for various reasons assigned the symbols $\alpha\varkappa^2\xi^2$ and $\alpha\varkappa^2\xi^4$, and of which they had confidently predicted the densities (or weights of the units) on their hydrogen standard to be respectively 29 and 45. When, however, their best experimentalists came to take the density of these gases, this density was found to be in each case only half that at which it had been estimated, namely 14·5 in the case of $\alpha\varkappa^2\xi^2$, and 22·5 in the case of $\alpha\varkappa^2\xi^4$.

The difficulty was serious enough; for hitherto in the construction of the units of matter from acetylene $\alpha\varkappa^2$, oxygen $\xi^2$, hydrogen $\alpha$, and the other elements, the operations $\alpha$, $\varkappa$, $\xi$ . . . had been sufficient for every requirement, hundreds of things had been made with these tools. But if these statements were to be unreservedly accepted, these operations would be inadequate to the purposes of the science; for the symbols of the units of these substances, as expressed by the letters $\alpha$, $\varkappa$, $\xi$, would be $\alpha^{\frac{1}{2}}\varkappa\xi$, $\alpha^{\frac{1}{2}}\varkappa\xi^2$, necessitating in their construction the performance of an operation $\alpha^{\frac{1}{2}}$, of which they had no experience whatever.

The unit $\alpha\varkappa^2\xi^2$ is the unit of a substance which the Laputians had, but which we have not. It goes in our language under the name of the radical of formic acid. $\alpha\varkappa^2\xi^4$ is the symbol we assign to the unit of oxalic acid. I proceed to explain how the Laputians dealt with this question.

"The accuracy of our experimentalists," said they, "is beyond suspicion, and we accept all the statements of fact made by them. But there is one point as to which it is necessary to inquire. How do they know, as their objections imply, that the gases of which they have taken the densities are really homogeneous gases? If at the temperature at which the experiment has been made these gases are really split in two, it is not necessary to modify, in this sense, our views.

"Now similar impediments have frequently stood in the way of our theories. But wherever it has been possible to look into the matter, as in the case of the pentachloride of phosphorus, and even hydrated sulphuric acid, theory has come out triumphant. This case is not strictly similar to those previously dealt with, but yet is not so dissimilar as to lead us to look for a totally different account of the matter. That we should discover a new tool $a^{\frac{1}{2}}$ does not, we must confess, appear to us very likely. But that we should be able to do more, with our old tools $\alpha$, $\varkappa$, $\xi$, than we have hitherto done is by no means out of the question. For the sake of argument let us assume, then, that the unit $\alpha\kappa^2\xi^2$, our binoxide of acetylene, cannot exist as a gas, but is invariably resolved into the two units $\alpha\kappa\xi$ and $\kappa\xi$ according to the equation

$$\alpha\kappa^2\xi^2=(\alpha\kappa\xi+\kappa\xi).$$

"The weight of a unit of these mixed gases will be precisely that given by your experiments, namely 14·5; and, moreover, it appears to us that we can explain all that we, or you, know about the gas from this point of view.

"The two units jointly (and you know no more) are identical with a unit of acetylene and a unit of oxygen; thus

$$(\alpha\kappa\xi+\kappa\xi)=\alpha\kappa^2+\xi^2.$$

"Five units of hydrogen and two units of your gas are identical with and capable of being resolved into two units of $\alpha^2\kappa$ (marsh-gas) and two units of $\alpha\xi$ (water); thus

$$5\alpha+(\alpha\kappa\xi+\kappa\xi)=2\alpha^2\kappa+2\alpha\xi,$$

which equation is identical with the sum of the two equations

$$2\alpha+\alpha\kappa\xi=\alpha^2\kappa+\alpha\xi,$$

$$3\alpha+\kappa\xi=\alpha^2\kappa+\alpha\xi.$$

"If you ask us to explain the origin of the unit $\alpha\kappa^2\xi^4$ (oxalic acid) by the oxidation of the unit $\alpha\kappa^2\xi^2$, we may go further in the same direction. We have

$$(\alpha\kappa\xi+\kappa\xi)+\xi^2=(\alpha\kappa\xi^2+\kappa\xi^2).$$

"Now the weight of a unit of the mixed gases $\alpha\kappa\xi^2$ (formic acid) and $\kappa\xi^2$ is precisely that which you have assigned to the unit $\alpha\kappa^2\xi^4$, namely 22·5.

"It is easy to render an account, on this assumption, of the chemical properties of this unit.

"Thus a unit of $\alpha\kappa^2\xi^4$ (oxalic acid), together with a unit of water, is identical with the unit $\alpha\kappa\xi^2$ (formic acid) and the unit $\alpha\kappa\xi^3$ (hydrated carbonic acid); thus

$$(\alpha\kappa\xi^2+\kappa\xi^2)+\alpha\xi=\alpha\kappa\xi^2+\alpha\kappa\xi^3.$$

"Again, three units of $\alpha\kappa^2\xi^4$, together with two units of water, are identical with one unit $\alpha\kappa^2\xi^2$ and four units $\alpha\kappa\xi^3$; thus

$$3(\alpha\kappa\xi^2+\kappa\xi^2)+2\alpha\xi=(\alpha\kappa\xi+\kappa\xi)+4\alpha\kappa\xi^3."$$

When the difficulty was pressed home upon the Laputian philosophers, that they were mere dreamers, for that their chemists had worked with these gases for more than 100 years and had never detected the presence in them of any one of these imaginary existences $\kappa\xi$, $\kappa\xi^2$, $\alpha\kappa\xi$, $\alpha\kappa\xi^2$, they replied that while it was true that many interesting objects lay upon the surface, so that anybody could see them, there were others not to be found unless they were specially looked for, and that no chemist had ever yet looked for these things. Also the difficulties in the way of their separation might be very great. The densities of these gases would be so nearly the same that it would be hopeless to attempt to separate them by diffusion. As to the similarity, up to a certain point, of the chemical properties of these substances $\alpha\varkappa\xi$ and $\varkappa\xi$ which it was necessary to assume, this need be no matter of surprise at all, for chemical substances (as everybody knew) existed in groups characterized by such similarities. When it was further urged that it would be very odd for such things to be found there and nowhere else in the world, they replied that on this point they really were incompetent to judge, but that, at any rate, many things as strange had more than once happened before, as in the case of the elements cæsium, rubidium, and lithium, three out-of-the-way things found all together in one mineral water, and the mineral gadolinite, which presented many curious problems which no one had yet fathomed. However, they added, "We always measure out our belief according to the evidence, and you are mistaken if you imagine us to be speaking over-confidently on such a subject."

Now we certainly cannot say that any Laputian chemists were placed in the position or reasoned in the way we have described; but we may say that if they had been placed in this position and reasoned thus they would have reasoned correctly, and their reasoning would have been fully justified by facts.

This argument is greatly strengthened by the numerous analogies connecting the elements $\nu$ and $\varkappa$. It is impracticable at this stage of our inquiries to bring these properly forward; but the reader will hereafter have the conclusion forced upon him that the element $\nu$ is, so to say, but a less potent carbon. At any rate, pursuing this train of thought, I have, as I must frankly confess, been brought to the opinion that by far the most probable explanation to be given of the anomalies presented by the densities of the binoxide and tetroxide of nitrogen is that the two gases which pass under this name are not homogeneous gases at all, but in each case are constituted of two gases which, taken together, are made up of the matter of oxygen and nitrogen, but which separately are not so made up. This is undoubtedly a speculation; but it is not a mere speculation, but one founded upon reasonable grounds, which explains difficulties for which no other explanation has ever been propounded, and suggests experiments by which it may be verified or disproved. I proceed to another subject.

The analytical construction of the symbols of the units of matter can be regarded only as preliminary to the consideration of a more complicated problem, namely, how these units are transformed in the processes of chemical change. On this subject there has been much speculation; but it is not going beyond the truth to say that no general theory of the nature of chemical events has yet been devised which will bear the slightest criticism or which is even intelligible. This question also we shall here consider from the analytical point of view afforded by the methods of this Calculus. "The mental," or, as it may be better termed, the "theoretical" analysis of a complex phenomenon into its elements is (it has been truly said) the first step of inductive inquiry[*]. This theoretical analysis of any object of our study is effected when we replace that thing by some system of things, the result of which, taken together, is equivalent to the

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[*] J. S. MILL, vol. i. c. vii. 1, p. 437; and also Contents to the same, p. xiv (ed. 1843).

result produced by the original object considered. Thus in mechanics the fundamental theorem of the parallelogram of forces is an analytical method, by which it is demonstrated that a force of a given magnitude operating in a given direction may always be theoretically resolved, in an infinite variety of ways, into two or more forces of certain specified magnitudes operating in certain specified directions, the result of which, taken together, is in magnitude and direction identical with the result of the force thus analyzed. Or, again, take the case of the analysis of vibratory movements. Defining the vibration of a point as "motion in a curve which returns into itself with a velocity which is always the same at the same point of the curve," it may be demonstrated that any given vibration of a point in a plane may be analyzed into two component rectilinear vibrations in an infinite variety of ways, and that every vibration of a point, whether plane or not, may be similarly resolved into three component rectilinear vibrations*.

Now precisely as forces are compounded of forces, as vibrations are compounded of vibrations, so chemical events are compounded of chemical events; and from the point of view of this Calculus the theoretical analysis of a chemical event consists in the theoretical determination of a certain special system or systems of such events, the total result of which, as regards the transformations of ponderable matter, is identical with that of the original event.

When we say that a chemical event is thus constituted of other events into which it may be resolved, the question arises, what view we take of the nature of such an event to justify this statement, and of what kind of events it is to be regarded as made up. The reply is given through the peculiar representation of a chemical event, afforded by the method of this Calculus, by which this fundamental conception is suggested. This representation is brought under our notice through the development of the method itself when we express dynamical facts by it, and consider how we are to reason upon them through its instrumentality. I shall therefore treat the subject in this natural order, considering first the construction of the "Organon" or Instrument of reasoning, then the ideas suggested to us by that Instrument, and to which that Instrument is applicable. When the nature of a chemical event has been thus defined so that we clearly see what we have to do, the further inquiry lies before us of the theoretical solution of the problem, namely, given a chemical event, how are we to determine the events of which that event is compounded? This problem may in all cases be solved. The solution is effected by means of a peculiar theorem, which occupies in theoretical chemistry a position analogous to that held in theoretical mechanics by the theorem of the parallelogram of forces. Lastly, I shall give examples of the application of this theorem to actual events and things, when the chemist will have an opportunity of estimating the bearing of this theory upon facts and its practical utility.

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* 'Acoustics, Theoretical,' by W. F. Donkin. Oxford, 1870. Chapter III. On the Composition of Vibrations.

## SECTION I.

(1) Our total information as to the identical relation of ponderable matter is comprised in a system of equations constituted upon the principles explained in Part I. Section VI. When we are adequately impressed with this important truth a chemical equation becomes a study of transcendent interest, and we are led to consider in a new light the purport and significance of its algebraical properties. Now even the science of Algebra itself has been defined as consisting but in the "analysis of equations," from which new truths are continually in process of evolution, and to the study of which all other objects of the science may be regarded but as accessory and subordinate. The same is emphatically true of the algebra of chemistry; and the most essential and characteristic feature of the Chemical Calculus, by which it is fundamentally discriminated from other modes of considering the science, is that in it we do not, as in the atomic theory, reason by the intervention of material images, but, setting aside all preconceived ideas, we base our arguments upon the equations themselves, and elicit from them, by the application of algebraical processes, the laws and principles which they implicitly contain.

Now I have been very unwilling to introduce irrelevant matter into a complicated subject; and in the first part of this Memoir, at the risk of some misapprehension, I have entirely confined myself to the consideration of questions the determination of which was essential to the end immediately in view, the "construction," namely, "of chemical symbols." But before proceeding further with the subject, it is necessary to recur to the fundamental principles of the method, and to discuss a question referred to in Part I. (Section IV. (5)), and there postponed, namely, under what conditions the operations of algebraical multiplication and division may be performed upon chemical equations.

The nature of such an equation, and the principles on which such equations are to be constructed, have been fully explained (Part I. Section V.), and it is only necessary to remind the reader that by a chemical equation is here meant an equation of the form

$$v=m\phi+m'\phi_1+m''\phi_2+m'''\phi_3+\ldots=0,$$

where

$$\phi=a^p b^{p_1} c^{p_2}\ldots,$$

$$\phi_1=a^q b^{q_1} c^{q_2}\ldots,$$

$$\phi_2=a^r b^{r_1} c^{r_2}\ldots,$$

$\phi, \phi_1, \phi_2 \ldots$ being the symbols of the units of matter, $a, b, c \ldots$ the symbols of simple weights (Section I. (8)), and $p, p_1, p_2 \ldots q, q_1, q_2 \ldots r, r_1, r_2 \ldots$ positive integers, and $m, m', m'', m''' \ldots$ numerical symbols, positive or negative, satisfying the conditions afforded by the system of indeterminate equations:

$$
\begin{aligned}
mp\ +m'q\ +m''r\ + \quad &=0,\\
mp_1+m'q_1+m''r_1+ \quad &=0,\\
mp_2+m''q_2+m'''r_2+ \quad &=0,\\
\ldots\ldots\ldots\ldots &\\
\ldots\ldots\ldots\ldots &
\end{aligned}
$$

It will be sufficient to consider the above question in relation to the fundamental equation (Part I. Section IV. (1))

$$xy=x+y, \quad \ldots\ldots\ldots\ldots \quad (1)$$

which affords the simplest example of a chemical equation. If we proceed to deal with this equation with the same freedom as with a numerical equation, and to multiply each member of this equation, not by a numerical symbol $m$ (which is permissible), but by a chemical symbol $v$, we have

$$vxy=vx+vy, \quad \ldots\ldots\ldots\ldots \quad (2)$$

a conclusion which is obviously false; for, interpreting the equation consistently with the principles on which these symbols are constructed, we are informed by it that the unit of matter symbolized as $vxy$ is identical, as regards "weight" or matter, with the aggregate of the two units symbolized as $vx$ and $vy$; and further, if we apply to equation (2) the principle laid down in equation (1), we have $vxy=v+xy$, $vx=v+x$, $vy=v+y$, and, as a final conclusion, $v=2v$. Again, taking the equation $x^2y=x+xy$, which is true, dividing both members of the equation by the chemical symbol $x$, we are led to an absurd conclusion, namely, that

$$xy=1+y, \quad \ldots\ldots\ldots\ldots \quad (3)$$

that is to say, that the matter symbolized as $xy$ is always identical with the matter of an empty unit of space, symbolized as 1, and the matter symbolized as $y$. But it is only necessary to observe that the equations (2) and (3) to which we are thus led,

$$
\begin{gathered}
vxy=vx+vy,\\
xy=1+y,
\end{gathered}
$$

are not chemical equations at all, for they do not satisfy the fundamental conditions afforded by the system of indeterminate equations previously referred to,

$$
\begin{gathered}
mp+m'q+m''r+\ldots 0,\\
\ldots\ldots\ldots
\end{gathered}
$$

In short, if we inconsiderately deal with these equations without reference to the principles on which they have been constructed, precisely as though they were numerical equations, we are not only involved in a tissue of absurdities and contradictions, but are compelled to introduce into the system of chemical equations, equations which do not belong to that system at all, and which could never arise from the expression by means of equations of chemical facts.

It is impossible for a false or absurd result to be the consequence of the application to a true statement of a correct principle of reasoning; and we are thus led to the singular conclusion that, although up to a certain point the analogy is perfect between the properties of chemical and numerical symbols, here this analogy terminates and ceases to be a correct guide; and we are apparently forced to admit that in the treatment of chemical equations no processes exist equivalent to the algebraical processes of multiplication and division. Now, first, we may observe that such an imperfect calculus might really exist, logically correct, but with limited methods; for there is nothing either in the nature of symbolical algebra or in the distributive and commutative laws,

$$x(y+z)=xy+xz,$$

$$xy=yx,$$

through which the symbols of chemical operations are related to the symbols of numbers (Part I. Section II. (4) and (5)), to necessitate the application of these processes to equations. Indeed in the actual applications of algebra perfectly similar restrictions are found to exist. The symbols of certain operations do not satisfy the commutative law, and we cannot infer from the equation

$$vxy=vx+vy$$

that (whatever be the interpretation of the symbols)

$$xy=x+y.$$

Thus, for example, in the algebra of logic, while the operation of algebraical multiplication, as applied to equations, is permissible, the performance of the operation of division upon logical equations is absolutely prohibited. Thus we cannot infer from the logical equation

$$zy=zx$$

that $y=x$, which would be tantamount to asserting that because those members of a class $y$ which possess a certain property $z$ are identical with those members of a class $x$ which possess the same property, therefore the class $y$ is identical with the class $x$ [BOOLE, 'Laws of Thought,' p. 36], an inference obviously erroneous. But even in the algebra of quantity we cannot always perform the operation of division upon equations where such division is formally possible. From the equation $x(x-a)=0$ we cannot infer that $x=0$. The operation of division can only be thus performed by the exclusion of certain symbols upon the assumption that 0 and $\frac{1}{0}$ are not values of the symbol by which we divide. Hence the circumstance that such processes are inadmissible does not detract in any degree from the truth, reality, or logical perfection of an algebraical system. Nevertheless it may be admitted with perfect consistency that a calculus from which such important processes were entirely excluded, and in which the processes of addition and subtraction were the only processes practically available, would be of very limited utility, and could only be regarded as a rudimentary and imperfect system, presenting but a remote resemblance to the extended methods of arithmetical algebra. Such a system is really (although not explicitly and avowedly) presented to us in our actual chemical notation. No algebraical operation has ever as yet been properly performed upon chemical symbols except the operations of addition and subtraction. The great convenience, the brevity and suggestiveness of the notation has led to the universal adoption of the apposition of letters as the expression of what is termed chemical combination. Thus HHO, $H_2O$, $H^2O$ (for each of these expressions is in use) represent to us the molecule of water as discriminated from the aggregate of hydrogen and oxygen, $H+H+O$. But these expressions are employed under protest and are algebraically lifeless, for no use is made of them; and, as far as any definite processes of reasoning are concerned in which such symbols are engaged, the molecule of water might with equal advantage have been expressed by the letter A.

(2) We shall now proceed to consider how chemical equations through the application of the principles of this Calculus may be brought into a form adapted to algebraical treatment.

First, it is to be observed that although the processes equivalent to multiplication and division are not universally applicable to chemical equations, yet neither, on the other hand, is it true that these processes are universally inapplicable. There are certain cases in which we may either multiply or divide a chemical equation by a chemical symbol, and the result shall be both interpretable and true. These cases are easily discriminated. Let us take the general equation given, Part I. Section V. (10),

$$v=m\phi+m'\phi_1+m''\phi_2+m'''\phi_3+\ldots=0,$$

and multiply the equation by the factor $e^t$, where $e$ is a chemical symbol and $t$ a number, we have then

$$me^t\phi+m'e^t\phi_1+m''e^t\phi_2+m'''e^t\phi_3+\ldots=0;$$

and then, in addition to the indeterminate equation

$$\begin{array}{ll} mp+m'q+m''r+ & =0 \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot & \end{array}$$

we have another equation to be satisfied, namely

$$mt+m't+m''t+m'''t+ \quad =0,$$

whence, by dividing by $t$,

$$m+m'+m''+m'''+ \quad =0.$$

Now if this equation be true, we are led into no error by effecting the multiplication in question. That is to say, we are justified in multiplying a chemical equation by any chemical factor of the form $e^t$ and by any product of such factors, if the sum of the numerical coefficients in that equation be equal to zero, but not otherwise.

Similarly, in the above equation, let $\phi$, $\phi_1$, $\phi_2$, $\phi_3$ . . . have a common measure $a^t$, so that $p=t+s$, $q=t+k$, $r=t+l$ . . .,

$$\begin{aligned} \phi &= a^{t+s}b^{p_1}c^{p_2}\ldots = a^{t+s}\Theta, \\ \phi_1 &= a^{t+k}b^{q_1}c^{q_2}\ldots = a^{t+k}\Theta_1, \\ \phi^2 &= a^{t+l}b^{r_1}c^{r_2}\ldots = a^{t+l}\Theta_2, \\ &\ldots\ldots\ldots; \end{aligned}$$

then

$$v=ma^{t+s}\Theta+m'a^{t+k}\Theta_1+m''a^{t+l}\Theta_2+\ldots$$

Now if we divide this equation by $a^t$, we have

$$ma^s\Theta+m'a^k\Theta_1+m''a^l\Theta_2+\ldots=0;$$

and if this equation be true,

$$ms+m'k+m''l+\ldots=0.$$

But we have from the original equation

$$m(t+s)+m'(t+k)+m''(t+l)+\ldots=0;$$

whence, subtracting,

$$m+m'+m''+m'''+\ldots=0.$$

That is to say, we are justified in dividing a chemical equation by any common measure of the symbols by which the equation is expressed of the form $a^t$ if the sum of the numerical coefficients in that equation be equal to zero, but not otherwise. An equation in which this condition is satisfied will be termed a "normal" chemical equation, being an equation to which the rules of algebra are applicable. An equation in which this condition is not satisfied will be termed an "abnormal" chemical equation, being an equation to which these rules cannot be applied.

If we open a chemical treatise and examine the equations expressive of actual results, we cannot fail to be impressed by the circumstance that numerous equations satisfy this condition and are of the "normal" form. For example,

$$2\alpha\chi=\alpha+\alpha\chi^2, \tag{1}$$

$$\alpha^2\phi\chi^3\xi+3\alpha\xi=3\alpha\chi+\alpha^2\phi\xi^4, \tag{2}$$

$$\alpha^3\varkappa^2\xi+\xi^2=\alpha\xi+\alpha^2\varkappa^2\xi^2, \tag{3}$$

$$2\alpha^2\nu+\alpha^5\varkappa^6\xi^4=2\alpha^3\varkappa^2\xi+\alpha^3\nu^2\varkappa^2\xi^2, \tag{4}$$

$$\alpha^4\varkappa^3\xi^3+3\alpha^2\varkappa^2\xi^2=3\alpha\xi+\alpha^7\varkappa^9\xi^6, \tag{5}$$

$$\alpha^3\varkappa^3\xi^3+2\alpha\omega=\alpha^3\varkappa^3\xi^2+\alpha\xi+\alpha\omega^2, \tag{6}$$

$$\alpha^7\varkappa^6\xi^6+11\alpha\omega=\alpha^7\varkappa^6\omega+6\alpha\xi+5\alpha\omega^2, \tag{7}$$

$$5\alpha+\alpha\omega^2\xi^5=5\alpha\xi+\alpha\omega^2. \tag{8}$$

These equations have been selected at random. They indicate to us the following facts:—

(1) Two units of hydrochloric acid are identical with a unit of hydrogen and a unit of chlorine.

(2) A unit of oxychloride of phosphorus and three units of water are identical with three units of hydrochloric acid and a unit of phosphoric acid.

(3) A unit of alcohol and a unit of oxygen are identical with a unit of water and a unit of acetic acid.

(4) Two units of ammonia and a unit of oxalic ether are identical with two units of alcohol and a unit of oxamide.

(5) A unit of glycerine and three units of acetic acid are identical with three units of water and one unit of triacetine.

(6) A unit of lactic acid and two units of hydriodic acid are identical with a unit of propionic acid, a unit of water, and a unit of iodine.

(7) A unit of mannite and eleven units of hydriodic acid are identical with a unit of iodide of hexyl, six units of water, and five units of iodine.

(8) Five units of hydrogen and a unit of pentoxide of iodine are identical with five units of water and a unit of iodine.

Such examples might be greatly multiplied. Equations possessing this peculiar property are not due to accident, but to the simplicity of natural laws. In the chemical metamorphoses of which they express the results there is no change of gaseous volume.

It would be by no means difficult thus to make a system of carefully selected equations to which, as satisfying the condition given above, the processes of algebra would be truly applicable. That such a system should be possible, that a system of "normal" equations, subordinate to algebraical laws, should actually be found in the midst, so to say, of an abnormal system, is undoubtedly a most striking and suggestive fact. But nevertheless, so far as any realization of the objects of a calculus are concerned, this circumstance, if taken alone, is totally inoperative. The result of constituting such a system would be to divide chemical equations into two classes—a class with which we could really deal algebraically, and a class with which we could not so deal; and corresponding to these classes we should have, as will presently be seen, two systems of phenomena—a system which we could realize, study, and comprehend, and a second system absolutely unintelligible to us. These anomalies, however, may be completely removed by a mathematical transformation of the equation, to which I must now request the attention of the reader.

(3) It has been remarked (I. Sec. IV. 3), where the meaning and properties of the chemical symbol 1 are under discussion, that any number of numerical symbols 0, 1, 2, 3 may be added to a chemical function without affecting the interpretation of that function as regards "weight." This may be inferred from the fundamental equation $xy=x+y$, which equation becomes if $y=1$, $x=x+1$. It is also an immediate consequence of the interpretation assigned in this Calculus to that symbol, which regarded as a symbol of "weight" (being the symbol of an empty unit of space) is the symbol of "no weight," and regarded as the symbol of an operation is the symbol of (if we may so say) "taking the unit of space as it is" without performing upon it any operation resulting in "weight"

or matter. So that, as regards "weight" or matter, our conception of the "weight" or matter of a unit of hydrogen symbolized as $\alpha$ is precisely the same thing as, and cannot be discriminated from, our conception of the "weight" or matter of a unit of hydrogen and the "weight" or matter of an empty unit of space. Hence as long as "weight" or matter is the only subject under our consideration, we may assert with perfect truth that $\alpha=\alpha+1$, this assertion being strictly analogous to the arithmetical assertion that $1=1+0$. In this peculiar property of the chemical symbol 1 is supplied to us the means of remedying the imperfection of chemical equations and of so changing their form, without affecting their interpretation, as to bring these equations within the recognized domain of algebra. For if in any chemical equation $v=0$ the sum of the numerical coefficients be not equal to zero, it is in our power to make that sum equal to zero, by the addition to the function $v$ of the necessary number of numerical symbols affected by the proper sign; and when this transformation is effected, we may operate upon the equation with confidence and security by every process of algebra—the result being on the one hand greatly to enlarge our powers, and on the other to prohibit inadmissible processes.

Take for example the equation discussed, Part I. Sec. VII. (8),

$$2\alpha^2\nu=3\alpha+\alpha\nu^2. \quad \ldots \quad (1)$$

This equation is "abnormal." We should be tempted from the form of the equation to consider the result of dividing by the symbol $\alpha$, which leads to an equation

$$2\alpha\nu=3+\nu^2,$$

an equation external to the system of chemical equations, the conditions previously referred to not being satisfied in that equation. But this equation, when rendered "normal" on the principles just laid down, is

$$2+2\alpha^2\nu=3\alpha+\alpha\nu^2. \quad \ldots \quad (2)$$

Now this equation, as regards any assertion made by it in relation to "weight," has precisely the same meaning as the equation from which it is derived; for it is perfectly immaterial whether we say that two units of ammonia are identical as regards "weight" with three units of hydrogen and one unit of nitrogen, or whether we say that two units of ammonia and two "units of space" are thus identical with three units of hydrogen and one unit of nitrogen. But as regards their algebraical properties the equations are fundamentally different; for on the latter equation (2) the performance of the operation of division by the symbol $\alpha$ is impossible, and the result of the transformation of the equation is absolutely to prohibit this operation. But, on the other hand, we may now multiply this equation by any chemical symbol, and the resulting equation will necessarily be interpretable and a proper subject for our consideration. As, for example, if we multiply both sides of the equation by the symbol $\xi^2$, the equation becomes

$$2\xi^2+2\alpha^2\nu\xi^2=3\alpha\xi^2+\alpha\nu^2\xi^2,$$

an equation in which the requisite conditions are satisfied.

The following examples will sufficiently illustrate the way in which this principle is to be applied for the construction of "normal" equations:—

$$2\alpha+\xi^2=2\alpha\xi, \quad \ldots \tag{1}$$

$$4\alpha^2\varphi=6\alpha+\alpha^2\varphi^4, \quad \ldots \tag{2}$$

$$\delta+\alpha\chi^2=\delta\alpha\chi^2, \quad \ldots \tag{3}$$

$$\alpha^2\varkappa^2+\alpha\beta^2=\alpha^3\varkappa^2\beta^2, \quad \ldots \tag{4}$$

$$\alpha\varkappa^2\xi^4=\alpha\varkappa\xi^2+\varkappa\xi^2, \quad \ldots \tag{5}$$

$$\alpha\xi+\varkappa\xi=\alpha\varkappa\xi^2, \quad \ldots \tag{6}$$

$$\alpha^2\varkappa^2\xi^2=\alpha^2\varkappa+\varkappa\xi^2, \quad \ldots \tag{7}$$

$$\alpha^4\varkappa^2\xi^2\nu=2\alpha\xi+\alpha^2\varkappa^2\nu, \quad \ldots \tag{8}$$

$$\alpha^2\varkappa^2\nu+2\alpha=2\alpha^4\varkappa^2\nu, \quad \ldots \tag{9}$$

$$\alpha^2\varkappa^2\xi^2\chi+2\alpha^2\nu=\alpha^3\nu\chi+\alpha^3\nu\varkappa^2\xi^2, \quad \ldots \tag{10}$$

$$\alpha^3\varkappa^4\xi^4+2\alpha^3\varphi\chi^5=2\alpha^2\varphi\chi^3\xi+2\alpha\chi+\alpha^3\varkappa^4\xi^2\chi^2, \quad \ldots \tag{11}$$

$$3\alpha^3\varkappa^7\xi+2\alpha^2\nu=\alpha^{10}\nu^2\varkappa^{21}+3\alpha\xi, \quad \ldots \tag{12}$$

$$2\alpha^4\varkappa^4\xi^2+3\xi^2=2\alpha\xi+2\alpha^3\varkappa^4\xi^4, \quad \ldots \tag{13}$$

$$\alpha^3\nu^2\varkappa\xi+\alpha\xi+3\alpha\chi^2=\varkappa\xi^2+\alpha\nu^2+6\alpha\chi, \quad \ldots \tag{14}$$

$$\alpha^3\nu^2\varkappa\xi+2\alpha\nu\xi^2=2\alpha\nu^2+\varkappa\xi^2+3\alpha\xi. \quad \ldots \tag{15}$$

The above equations are strictly true, and are the records of actual observations. They are thus to be interpreted:—

(1) Two units of hydrogen and one unit of oxygen are identical with two units of water.

(2) Four units of phosphide of hydrogen are identical with six units of hydrogen and a unit of phosphorus.

(3) A unit of mercury and a unit of chlorine are identical with a unit of protochloride of mercury.

(4) A unit of ethylene and a unit of bromine are identical with a unit of bromide of ethylene.

(5) A unit of oxalic acid is identical with a unit of formic acid and a unit of carbonic acid.

(6) A unit of water and a unit of carbonic oxide are identical with a unit of formic acid.

(7) A unit of acetic acid is identical with a unit of marsh-gas and a unit of carbonic acid.

(8) A unit of acetate of ammonia is identical with two units of water and a unit of aceto-nitrile.

(9) A unit of aceto-nitrile and two units of hydrogen are identical with two units of ethylamine.

(10) A unit of monochloracetic acid and two units of ammonia are identical with a unit of chloride of ammonium and a unit of glycocoll.

(11) A unit of succinic acid and two units of pentachloride of phosphorus are identical with two units of oxychloride of phosphorus and two units of hydrochloric acid and a unit of chloride of succinyl.

(12) Three units of benzoic aldehyde and two units of ammonia are identical with a unit of hydrobenzamide and three units of water.

(13) Two units of butyric acid and three units of oxygen are identical with two units of water and two units of succinic acid.

(14) A unit of urea and a unit of water and three units of chlorine are identical with a unit of carbonic acid, a unit of nitrogen, and six units of hydrochloric acid.

(15) A unit of urea and two units of nitrous acid are identical with two units of nitrogen, a unit of carbonic acid, and three units of water.

These equations, although true, are "abnormal," and cannot be dealt with algebraically.

In the normal form they appear as follows:—

$$2\alpha+\xi^2=2\alpha\xi+1, \tag{1}$$

$$3+4\alpha^2\phi=6\alpha+\alpha^2\phi^4, \tag{2}$$

$$\delta+\alpha\chi^2=\delta\alpha\chi^2+1, \tag{3}$$

$$\alpha^2\varkappa^2+\alpha\beta^2=\alpha^3\varkappa^2\beta^2+1, \tag{4}$$

$$1+\alpha\varkappa^2\xi^4=\alpha\varkappa\xi^2+\varkappa\xi^2, \tag{5}$$

$$\alpha\xi+\varkappa\xi=\alpha\varkappa\xi^2+1, \tag{6}$$

$$1+\alpha^2\varkappa^2\xi^2=\alpha^2\varkappa+\xi^2, \tag{7}$$

$$2+\alpha^4\varkappa^2\xi^2\nu=2\alpha\xi+\alpha^2\varkappa^2\nu, \tag{8}$$

$$\alpha^2\varkappa^2\nu+2\alpha=2\alpha^4\varkappa^2\nu+1, \tag{9}$$

$$\alpha^2\varkappa^2\xi^2\chi+2\alpha^2\nu=\alpha^3\nu\chi+\alpha^3\nu\varkappa^2\xi^2+1, \tag{10}$$

$$2+\alpha^3\varkappa^4\xi^4+2\alpha^3\phi\chi^5=2\alpha^2\phi\chi^3\xi+2\alpha\chi+\alpha^3\varkappa^4\xi^2\chi^2, \tag{11}$$

$$3\alpha^3\varkappa^7\xi+2\alpha^2\nu=\alpha^{10}\nu^2\varkappa^{21}+3\alpha\xi+1, \tag{12}$$

$$2\alpha^4\varkappa^4\xi^2+3\xi^2=2\alpha\xi+2\alpha^3\varkappa^4\xi^4+1, \tag{13}$$

$$3+\alpha^3\nu^2\varkappa\xi+\alpha\xi+3\alpha\chi^2=\varkappa\xi^2+\alpha\nu^2+6\alpha\chi, \tag{14}$$

$$3+\alpha^3\nu^2\varkappa\xi+2\alpha\nu\xi^2=2\alpha\nu^2+\varkappa\xi^2+3\alpha\xi. \tag{15}$$

In the preceding examples the equations are rendered "normal" by the arbitrary application of a process, proved to be permissible according to the principles of the method—this process depending upon the circumstances that the interpretation of any chemical function is not (as regards "weight" or matter) affected by the addition to it or subtraction from it of the symbol 1. The application of this principle to the general equation

$$xy=x+y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

leads to the "normal" form of that equation,

$$1+xy=x+y.. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(4) We may also arrive at this last equation through the following considerations, which involve some important principles which will hereafter find a further application. Let A, $x, y$ be chemical symbols, and let

$$\mathrm{A}xy+\mathrm{A}ab=\mathrm{A}xb+\mathrm{A}ya;$$

then, from equation (1),

$$\mathrm{A}xy=\mathrm{A}+x+y, \qquad \mathrm{A}xa=\mathrm{A}+x+a,$$
$$\mathrm{A}ab=\mathrm{A}+a+b, \qquad \mathrm{A}yb=\mathrm{A}+y+b.$$

Now putting

$$v=\mathrm{A}xy+\mathrm{A}ab-\mathrm{A}xb-\mathrm{A}ya,$$

and substituting for $\mathrm{A}xy$, $\mathrm{A}ab$, $\mathrm{A}xa$, $\mathrm{A}yb$ the above values,

$$v=0.$$

But we have, from the distributive property of chemical symbols (established in I. Sec. II. 6),

$$v=\mathrm{A}(x-a)(y-b), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

whence

$$\mathrm{A}(x-a)(y-b)=0;$$

that is to say, the product of any two chemical factors of the forms $x-a, y-b$ is necessarily equal to nothing; and the further application of the same reasoning leads to the conclusion that the continued product of any number (not less than two) of factors of this kind is also nothing; that is,

$$\mathrm{A}(x-a)(y-b)(z-c)\ldots=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now, since these equations are always true, they are true when A, $a$, and $b$ are respectively put equal to 1, whence

$$(x-1)(y-1)=0$$

and

$$1+xy=x+y,$$

which is the equation referred to.

Now this equation, regarded as a numerical equation, is satisfied by the values $x=1, y=1$; and since this property is perfectly general, and every symbol of a chemical operation satisfies this condition, every chemical equation must necessarily be true when the prime factors by which it is expressed are severally or collectively put equal to 1.

(5) Now if this principle be applied to an abnormal equation, we are led to the assertion (Part I. Sec. IV. (3))

$$1(1)=2(1)=3(1)\ldots=n(1)=0;$$

or, suppressing the chemical symbol (1), $1=2=3\ldots=n=0$, which assertion is indeed not inconsistent with the use and interpretation of the chemical symbol (1), but is entirely at variance with the numerical interpretation of that symbol, so that the greatest possible divergence is apparently exhibited between the use which is made in this Calculus of that symbol and its arithmetical signification*. But if this same substitution be effected in a "normal" equation the anomaly disappears. No divergence is found between the arithmetical and chemical use of the symbol 1, and the equation is true *both* as an arithmetical and chemical equation; for from the very principle of its construction every "normal" equation must necessarily vanish when the prime factors in that equation are severally put equal to 1. So that the effect of making this substitution is simply to lead us to the assertion that $0=0$, which is an arithmetical as well as chemical truth; that is to say, every "normal" equation is true, not only as a "chemical equation," but also as an arithmetical equation for the only arithmetical value which the symbols can possibly assume, namely the value 1; and when this value is assigned to every prime factor by which the equation is expressed, the chemical equation is turned into an arithmetical equation, which is both interpretable and true. The reason of this is that the chemical symbol 1 is absolutely identical, as regards its algebraical properties, with the arithmetical symbol 1, although having a totally different interpretation from that symbol, a point which has been fully demonstrated and discussed in Part I. Sec. III., and also that a "normal" equation is subject to precisely the same rules of algebraical treatment as an arithmetical equation, as has been just demonstrated.

The real and valid character of the preceding reasoning will be evident from the following example, which illustrates the way in which a chemical function passes into a numerical function when chemical symbols assume the numerical value 1, and the inferences which may be thus drawn.

(6) In the first part of this memoir the symbols of the units of chemical substances are determined by the aid of certain indeterminate numerical equations, at which we arrive by the direct application to chemical equations of the fundamental properties of chemical symbols so frequently referred to $xy=x+y$ (I. Sec. VII.).

Take, for example, the equation given, Part I. Sec. VII. (8),

$$2\alpha^{m}\nu^{m_1}=3\alpha+\alpha^{n}\nu^{n_1};$$

from this equation we are able immediately to infer that

$$(\alpha^{m}\nu^{m_1})^2=\alpha^{3}.\,\alpha^{n}\nu^{n_1},$$

whence

$$2m=3+n, \qquad 2m_1=n_1.$$

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* The apparent paradox involved in this assertion may be removed by assigning a special symbol, $e^p$, to the unit of space, $p$ being a positive integer. But on investigating the properties of this symbol we should soon find that $e=e^p$, whatever be the value of $p$ (the value 0 included), and that, as we might always replace algebraically the symbol $e^p$ by the symbol (1), we were really dealing with the symbol 1 under another name. Such paradoxes, however, have no significance when the meaning of the expressions employed is properly understood. Thus DE MORGAN, in his 'Double Algebra' (ed. 1849, p. 114), speaking of the term addition as there employed by him, says, "*Nor is there, in one sense, the slightest objection to saying that* 12 *and* 12 *make* 10," an assertion quite as paradoxical (to say the least) as any here made.

But we may also proceed thus. Reducing the above equation to the "normal" form, we have

$$2+2\alpha^m\nu^{m_1}=3\alpha+\alpha^n\nu^{n_1}.$$

Putting $\nu=1$, we have

$$2+2\alpha^m=3\alpha+\alpha^n,$$

whence

$$\frac{\alpha(2\alpha^{m-1}-\alpha^{n-1}-1)}{\alpha-1}=2.$$

Since this equation is always true, it is true where $\alpha=1$. But the numerical value of the left-hand member of this equation, when $\alpha=1$, is $2(m-1)-(n-1)$; whence we have

$$2m-n-1=2,$$

and, as before,

$$2m=3+n;$$

again, putting $\alpha=1$, we have

$$2+2\nu^{m_1}=3+\nu^{n_1}$$

and

$$\frac{\nu^{m_1}-\nu^{n_1}}{\nu^{m_1}-1}=-1.$$

But the numerical value of $\frac{\nu^{m_1}-\nu^{n_1}}{\nu^{m_1}-1}$, when $\nu=1$, is $\frac{m_1-n_1}{m_1}$; whence we have, as before,

$$2m_1=n_1.$$

We thus arrive by the consideration of the numerical value of chemical symbols at the results previously inferred by an apparently different process. But the truth is that the very same principles are really employed in both methods, which differ only in the stage at which those principles are introduced.

The process here employed for the reduction of a chemical equation to the "normal" form, with the view of rendering that equation amenable to algebraical treatment, is strictly in conformity with the spirit of algebraical methods. The ordinary artifice for the solution of a quadratic equation is based upon a somewhat similar principle. A particular form of the symbol 0 is there added to the function, by which its value and interpretation are not affected, but which enables us to express the function by means of factors. But an illustration, perhaps still more in point, is the process familiar to mathematicians, by which an equation is rendered "homogeneous," which is effected by the introduction of an arbitrary factor with which we work precisely as with one of the real factors of the symbols which appear in the equation, and which, at the conclusion of the algebraical operations, to which the equation is submitted, is put equal to 1 and suppressed.

(7) It is, however, important to notice that although it is strictly true that if the

assertion made in the symbol = be limited to identity of "weight," $x=x+1$; so that, as regards this identity, the assertion made in the "normal" equation

$$1+2\alpha\xi=2\alpha+\xi^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

is the very same assertion as that made in the "abnormal" equation

$$2\alpha\xi=2\alpha+\xi^2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

it being perfectly immaterial, as before remarked, whether we say that the "weight" of a unit of space and the "weight" of two units of water is identical with the "weight" of two units of hydrogen and a unit of oxygen, or whether we simply say that the "weight" of two units of water is identical with the "weight" of two units of hydrogen and a unit of oxygen, just as it is immaterial whether we say that three and two are identical with five, or that nothing and three and two are identical with five, the selection among such expressions being dictated merely by algebraical convenience. Yet nevertheless a very real and appreciable distinction may be drawn between the two assertions, which from another aspect have by no means a strictly equivalent meaning. The former equation (1) necessarily includes every assertion which is made in the latter; but the latter (2) does not include every assertion which is made in the former. For if we extend our notion of identity to identity of "space" as well as of "matter," it is open to us to assign an interpretation to the symbol of identity = in equation (1) which cannot be assigned to that symbol in equation (2). A unit of space and two units of water are identical, both as regards "weight" and as regards the "space" occupied by that "weight," with two units of hydrogen and a unit of oxygen. But it is not true that two units of water are identical, both as regards "weight" and as regards the "space" occupied by that "weight," with two units of hydrogen and a unit of oxygen; for the space occupied by two units of water is 2000 cub. centims., but the space occupied by two units of hydrogen and a unit of oxygen is 3000 cub. centims. [I. Sec. I. (10, 11)]. Thence, in the reduction of a chemical equation to the "normal" form, we not only render the equation algebraically correct, but also extend and complete the idea expressed in the equation.

Now this conception of a perfect and complete identity, both as regards "space" and matter, cannot arise so long as we confine our inquiries to the composition of the units of ponderable matter, as considered in the first part of this Calculus, but has its origin exclusively in those dynamical inquiries which relate to the changes of matter. For this there is an obvious reason. In investigating the laws of gaseous combinations, which are the special objects of this method, a most essential point to be considered, whether from the point of view of the chemist or of the physical philosopher, is the relation of "matter" to "space;" and in the adequate consideration of a chemical change we are bound to take cognizance not only of changes in the composition of the "units of ponderable matter," but also of changes in the "bulk" of matter, and to find the means of entering these changes in our equations. The contrivance by which a chemical equation is rendered "normal" is a method of effecting this entry.

(8) According to the reasoning previously pursued we arrive at the numerical equation $2m=3+n$, $2m=n$, by successively assigning to the chemical symbols $\alpha, \nu$ the value 1, a value common to numerical and chemical symbols; but this method is but an illustration of a wider principle. We may lay down the rule, that if throughout a chemical equation we substitute the symbol of any one "simple weight" for any other "simple weight," that equation will still be true. This statement does not mean that the symbols of all simple weights have the same meaning, and that it is indifferent which we employ, or that the assertion made in the equation thus modified is the same as the assertion made in the original equation, but that if the original equation be true, the new assertion made in the equation thus modified is also true. We may compare the two members of a chemical equation to the two pans of a balance which is kept in equilibrium by a system of weights of 1 ounce, 2 ounces, 3 ounces, 1 lb., 2 lbs., 3 lbs., and the like in each pan, the same number of weights of each kind being in the two pans respectively. The equilibrium of the balance is not only unaffected by any change in the distribution (or arrangement) of the weights in each pan, but is also unaffected by the simultaneous removal from each pan of all the weights of the same kind, or by substituting weights weighing 1 lb. for the weights weighing 1 ounce, or any analogous substitutions. At first sight it would appear that the chemical symbol 1 should be excluded from the operation of this rule; for it is not the case that in the normal equation

$$1+xy=x+y,$$

where the symbol 1 appears we may write $x$ (although where $x$ appears we may write 1), and that the resulting equation should still be true. The reply to this difficulty is that, in order to arrive at the above equation, we have tacitly assumed the truth of the equation $1^p=1$, and that the equation in the above form is really imperfect. If, however, we render the equation homogeneous, remembering that $\varpi^p=1$, whatever be the value of $p$, we have for the complete equation

$$\varpi^2+xy=x\varpi+y\varpi,$$

to which the rule is applicable.

## SECTION II.—ON SIMPLE AND COMPOUND EVENTS.

(1) Through the transformation of equations, described in the last section, the Chemical Calculus is finally constituted as a symbolical method adapted to algebraical reasoning. I shall now proceed to consider, in the light of these principles, the nature of the fundamental conceptions through which we are to reason as to the phenomena of chemical change. Here, again, pursuing the method employed in the first part of this Calculus, I shall endeavour to assign a precise meaning to the terms employed, and accurately to define those conceptions. But in this case the notions themselves are so

mixed up with the symbolical forms in which they originate, that it would be inexpedient and indeed almost impracticable, to attempt to explain these conceptions except in reference to these forms, and these definitions will be given as the occasion arises.

A chemical equation has been hitherto regarded in this Calculus only as an assertion in regard to the identity of certain units of ponderable matter. But there is another and equally just point of view from which such an equation may be considered, namely, as the record of a "chemical event" or "metamorphosis;" and a chemical equation of the form $u=0$ may correctly be termed the symbol of such an event. For from the very principle on which such equations are constructed, the assertion that A is identical as regards "weight" with B necessarily implies that A has been chemically converted into B, or that B has been chemically converted into A, for otherwise we could not possibly have been aware of this identity, our only knowledge of the identical relation of matter being derived, as has already been fully explained (I. Sec. VI.), from the chemical transmutation of matter. And when we write down such equations as those given in the last section (Sec. I. (2)(3)), we are recording the various events of which we have become cognizant in the history of chemical metamorphosis.

The term "chemical event" will be here employed to denote the occurrence of any change whatever in the chemical composition of the units of matter of which the result is or may be expressed in an equation; and precisely as we speak of a "chemical operation" as the "simple weight" which is the result of that operation, so also may we refer to a "chemical equation" as the "chemical event" of which it symbolizes the result, and deal with it as representing the event itself. Such an event is here represented to us as in the strictest sense a "metamorphosis" or "change of form;" not that we venture to offer any material image, picture, or physical representation of that as yet inscrutable phenomenon, but we speak of it in this language because that change is indicated to us and adequately expressed by changes in the arrangement and structure of these symbolic forms, through which it is brought under the cognizance of our intelligence.

(2) Now just as the algebraical sum or aggregate (as we shall term it) of two or more equations itself constitutes an equation, so every collection or aggregate of chemical events constitutes an event. This is true whether those events are considered as occurring successively or simultaneously. An event of which the result is expressed by the algebraical sum of two or more equations will be termed "an aggregate," and will be spoken of as "the aggregate" of those two or more events expressed in those equations, and will be said to be "constituted" of those events. We are hence led to discriminate chemical events as "simple" and "compound" events.

*Definition.*—A "compound" event is an event which is regarded in the system of events under our consideration, whatever that may be, as "constituted" of two or more events, and "a simple event" is an event which in that system is not so regarded. But these terms necessarily have reference to some special mode of regarding and considering the phenomena, and apart from such considerations the terms are unmeaning;

for there is no property of a chemical phenomenon which compels us to regard it either from the one or the other point of view. In this respect the use made of the terms "simple event" and "compound event" is analogous to the use made in the first part of this memoir of the terms "simple weight" and "compound weight," a division coextensive with that into "distributed" and "undistributed" weights (I. Sec. I. 7, 8, 12). For not only are these terms dependent upon one another, a "simple weight" being defined as a "weight" which is not "compound," but they also have reference to some special system of phenomena considered from a definite point of view—a "distributed" weight being a weight which *in such a system of events* is resolved into two or more weights, or made up from such weights, and an "undistributed" weight being a weight which *in the same system of events* is *not* so resolved or so made up," there being nothing whatever in the properties of matter, apart from such considerations, to justify the use and application of such terms at all. Hence a "weight" which from one point of view and in one system of events is regarded as a "compound" and "distributed" weight, from another point of view and in another system of events may, with equal reason, be regarded as a "simple" or "undistributed" weight. These conceptions have their origin in the consideration of the chemical properties of matter, but may be transferred without any fundamental alteration of meaning from matter to phenomena, or "the changes of matter"—the symbol of a compound event being the symbol of one event which is "an aggregate" or collection of some other events, precisely as the symbol of the unit of hydrochloric acid $\alpha\chi$ is the symbol of one operation which is a combination of the two operations $\alpha$ and $\chi$. As it is essential that a correct appreciation should be made of what is here meant by a compound event, I shall proceed to give some examples of such events.

Example (1).—Let us suppose ourselves to be informed that a unit of binoxide of hydrogen is identical with a unit of hydrogen and a unit of oxygen, that is to say, that

$$1+\alpha\xi^2=\alpha+\xi^2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

this information could only be derived from the circumstance that the event of the transformation of a unit of binoxide of hydrogen into a unit of hydrogen and a unit of oxygen had, in some system of chemical metamorphoses, actually occurred, and the above equation informs us of all the circumstances of that event (as regards the identities of matter and space) which the event involves. Now if no other information than this, either real or theoretical, be laid before us, we can form but one judgment as to that event, namely, that it is a "simple event;" for we are supplied with no information which enables us to regard it as a "compound event," and a "simple event" is an event which, in relation to our information, is *not* compound. But let us also be informed of the two following circumstances, namely, that two units of binoxide of hydrogen are identical with two units of water and a unit of oxygen, and also that two units of water are identical with two units of hydrogen and a unit of oxygen. The events whence this information was derived are thus recorded:—

$$1+2\alpha\xi^2=2\alpha\xi+\xi^2, \quad \ldots \ldots \quad (2)$$

$$1+2\alpha\xi=2\alpha+\xi^2. \quad \ldots \ldots \quad (3)$$

It is now in our power to take a different view of the nature of the event,

$$1+\alpha\xi^2=\alpha+\xi^2. \quad \ldots \ldots \quad (1)$$

For if we add together the right-hand and left-hand members of equations (2) and (3) and divide the result by 2, we constitute equation (1); that is to say, this event (1) may be regarded as a compound event, which is the aggregate of the two events,

$$\tfrac{1}{2}+\alpha\xi^2=\alpha\xi+\tfrac{1}{2}\xi^2,$$

$$\tfrac{1}{2}+\alpha\xi=\alpha+\tfrac{1}{2}\xi^2;$$

for not only is equation (1) an algebraical inference from equations (2) and (3), but it is actually in our power to effect the chemical transformation of a unit of binoxide of hydrogen into a unit of hydrogen and a unit of oxygen by effecting two several transformations, namely, the transformation of a unit of binoxide of hydrogen into a unit of water and half a unit of oxygen, and also the transformation of a unit of water into a unit of hydrogen and half a unit of oxygen; and the result of these events (whether they occur successively or simultaneously, or in whatever order) is the transformation of a unit of binoxide of hydrogen into a unit of hydrogen and a unit of oxygen, and no other occurrence or event—it being quite immaterial as regards the result whether we first transform a unit of binoxide of hydrogen into a unit of water and half a unit of oxygen, and then take that unit of water and resolve it into a unit of hydrogen and half a unit of oxygen, or whether we begin by resolving a unit of water into a unit of hydrogen and half a unit of oxygen, and then resolve a unit of binoxide of hydrogen into a unit of water and half a unit of oxygen, or whether the two events occur together. In each case the final result is precisely the same, namely, the conversion of a unit of binoxide of hydrogen into a unit of hydrogen and a unit of oxygen, and nothing else.

Now as the equation

$$1+\alpha\xi^2=\alpha+\xi^2 \quad \ldots \ldots \quad (1)$$

is an inference which may be arrived at in many ways and from many systems of equations, so may we regard this event as "constituted" of other "events" in many ways. Thus, for example, a unit of chlorine and a unit of binoxide of hydrogen are identical with two units of hydrochloric acid and a unit of oxygen, and also two units of hydrochloric acid are identical with a unit of chlorine and a unit of hydrogen, whence

$$1+\alpha\chi^2+\alpha\xi^2=2\alpha\chi+\xi^2,$$

$$2\alpha\chi=\alpha+\alpha\chi^2.$$

The event (1) is also an aggregate of these two events. But these events, of which event (1) is constituted, may themselves be compound events, and that in various ways. Thus, for example, the event

$$1+\alpha\chi^2+\alpha\xi^2=2\alpha\chi+\xi^2$$

may be considered as the aggregate of the two events,

$$2\alpha\chi+\alpha\xi^2=\alpha\chi^2+2\alpha\xi.$$

$$1+2\alpha\chi^2+2\alpha\xi=4\alpha\chi+\xi^2;$$

in which case the event (1) will be the aggregate of three events—

$$2\alpha\chi+\alpha\xi^2=\alpha\chi^2+2\alpha\xi,$$

$$1+2\alpha\chi^2+2\alpha\xi=4\alpha\chi+\xi^2,$$

$$2\alpha\chi=\alpha+\alpha\chi^2.$$

Example (2).—Again, a unit of ammonia and three units of iodide of ethyl are identical with three units of hydriodic acid and a unit of triethylamine.

This event is thus expressed:

$$\alpha^2\nu+3\alpha^3\varkappa^2\omega=3\alpha\omega+\alpha^8\varkappa^6\nu.$$

Now this event may be regarded as the aggregate of the three following events:—

$$\alpha^2\nu+\alpha^3\varkappa^2\omega=\alpha\omega+\alpha^4\varkappa^2\nu,$$

$$\alpha^4\varkappa^2\nu+\alpha^3\varkappa^2\omega=\alpha\omega+\alpha^6\varkappa^4\nu,$$

$$\alpha^6\varkappa^4\nu+\alpha^3\varkappa^2\omega=\alpha\omega+\alpha^8\varkappa^4\nu.$$

Each of these events, too, may itself be regarded as a compound event; thus the event

$$\alpha^2\nu+\alpha^3\varkappa^2\omega=\alpha\omega+\alpha^4\varkappa^2\nu$$

may be considered as constituted of the following events:—

$$\alpha^3\varkappa^2\omega+\alpha\varkappa\nu\xi=\alpha^3\varkappa^3\nu\xi+\alpha\omega, \qquad \ldots\ldots\ldots\ldots \quad \text{(a)}$$

$$\alpha^3\varkappa^3\nu\xi+\alpha\xi=\alpha^4\varkappa^2\nu+\varkappa\xi^2, \qquad \ldots\ldots\ldots\ldots \quad \text{(b)}$$

$$\varkappa\xi^2+\alpha^2\nu=\alpha\xi+\alpha\varkappa\nu\xi. \qquad \ldots\ldots\ldots\ldots \quad \text{(c)}$$

For if (a) we take a unit of iodide of ethyl and a unit of cyanic acid and transform them into a unit of hydriodic acid and a unit of cyanic ether, and then (b) take a unit of cyanic ether and a unit of water and transform these into a unit of ethylamine and a unit of carbonic acid, and also (c) take a unit of ammonia and a unit of carbonic acid and transform these into a unit of water and a unit of cyanic acid (3), the final result of these successive events will be to transform a unit of ammonia and a unit of iodide of ethyl into a unit of hydriodic acid and a unit of ethylamine, which is the event in question.

Example (3).—The following equation expresses to us the transformation of two units of marsh-gas and a unit of oxygen into two units of methylic alcohol,

$$2\alpha^2\varkappa+\xi^2=2\alpha^2\varkappa\xi+1.$$

This event may be regarded as an aggregate of seven events, as follows:—

$$\begin{aligned}
\alpha^2\varkappa\ \ +\alpha\chi^2 &= \alpha^2\varkappa\chi+\alpha\chi,\\
\alpha^2\varkappa\ \ +\alpha\chi^2 &= \alpha^2\varkappa\chi+\alpha\chi,\\
\alpha^2\varkappa\chi+\alpha\xi &= \alpha^2\varkappa\xi+\alpha\chi,\\
\alpha^2\varkappa\chi+\alpha\xi &= \alpha^2\varkappa\xi+\alpha\chi,\\
2\alpha+\xi^2 &= 2\alpha\xi+1,\\
2\alpha\chi &= \alpha+\alpha\chi^2,\\
2\alpha\chi &= \alpha+\alpha\chi^2,
\end{aligned}$$

and has absolutely been realized by the synthesis of these events. But each of these events may itself be an aggregate. Take, for example, the event

$$\alpha^2\varkappa\chi+\alpha\xi=\alpha^2\varkappa\xi+\alpha\chi\,;$$

this result may be obtained by the synthesis of the two following events:—

$$\alpha^2\varkappa\chi+\alpha^2\varkappa^2\xi^2=\alpha\chi+\alpha^3\varkappa^3\xi^2, \quad \ldots\ldots\ldots \tag{a}$$

$$\alpha^3\varkappa^3\xi^2+\alpha\xi=\alpha^2\varkappa^2\xi^2+\alpha^2\varkappa\xi. \quad \ldots\ldots\ldots \tag{b}$$

For if we take (a) a unit of chloride of methyl and a unit of acetic acid and transform them into a unit of hydrochloric acid and a unit of acetate of methyl, and then (b) take that unit of acetate of methyl and a unit of water and transform them into a unit of acetic acid and a unit of methylic alcohol, the final result of these successive transformations will be the transformation of a unit of chloride of methyl and a unit of water into a unit of alcohol and a unit of hydrochloric acid.

Example (4).—Again, let the event considered be the synthetical construction of methylic alcohol from carbon, hydrogen, and oxygen according to the following equation,

$$2\varkappa+4\alpha+\xi^2=2\alpha^2\varkappa\xi+5. \quad \ldots\ldots\ldots \tag{1}$$

Now this event may be regarded as an aggregate of the three following events, the first of which is the "aggregate" considered in the last example, the latter events being repeated twice,

$$(2) \qquad 2\alpha^2\varkappa+\xi^2=2\alpha^2\varkappa\xi+1,$$

$$(3) \qquad 2\alpha+\varkappa=\alpha^2\varkappa+2.$$

Again, the second of these two events may be regarded as constituted of the following events:—

(1) Formation of acetylene from carbon and hydrogen,

$$2\varkappa+\alpha=\alpha\varkappa^2+2.$$

(2) Formation of olefiant gas from acetylene and hydrogen,

$$\alpha\varkappa^2+\alpha=\alpha^2\varkappa^2+1.$$

(3) Formation of alcohol from olefiant gas and water,

$$\alpha^2\kappa^2+\alpha\xi=\alpha^3\kappa^2\xi+1.$$

(4) Oxidation of alcohol with the formation of acetic acid and water,

$$\alpha^3\kappa^2\xi+\xi^2=\alpha\xi+\alpha^2\kappa^2\xi^2.$$

(5) Decomposition of acetic acid into marsh-gas and carbonic acid,

$$1+\alpha^2\kappa^2\xi^2=\alpha^2\kappa+\kappa\xi^2.$$

(6) Resolution of carbonic acid into carbon and oxygen,

$$1+\kappa\xi^2=\kappa+\xi^2.$$

For if we write down these equations in order, and add the corresponding members of those equations, we arrive at the equation required,

$$\begin{gathered}2\kappa+\alpha=\alpha\kappa^2+2,\\ \alpha\kappa^2+\alpha=\alpha^2\kappa^2+1,\\ \alpha^2\kappa^2+\alpha\xi=\alpha^3\kappa^2\xi+1,\\ \alpha^3\kappa^2\xi+\xi^2=\alpha\xi+\alpha^2\kappa^2\xi^2,\\ 1+\alpha^2\kappa^2\xi^2=\alpha^2\kappa+\kappa\xi^2,\\ 1+\kappa\xi^2=\kappa+\xi^2.\end{gathered}$$

Now we have seen in the last example that the event (2) may be considered as constituted of 8 events, and the event (3) as constituted of 6 events, each of which is repeated twice in the "compound event" (1), which therefore is to be regarded as constituted of 20 distinct "occurrences" or "events." This last example is derived, with certain modifications, from the work of the distinguished chemist BERTHELOT, 'Chimie organique fondée sur la Synthèse,' where numerous similar illustrations of the aggregation of phenomena may be found. "C'est ainsi," says BERTHELOT, "que l'on parvient à la synthèse d'un alcool par une succession régulière de réactions définies" (Chimie organique fondée sur la Synthèse, ed. 1860, vol. i. p. 97).

The synthesis of an alcohol thus effected by BERTHELOT is undoubtedly brought about by the aggregation of phenomena, and so far affords an illustration of a compound event. But this aggregate of phenomena is an accidental collection of events selected to attain a particular end, which, although concurring to produce the result in question, are yet marked by no regular order or sequence and are characterized by no common properties. Such aggregates are not to be confounded with the aggregates subsequently considered in this Calculus.

## SECTION III.—ON THE "CAUSES" OF CHEMICAL EVENTS.

Having thus arrived at the conception of the nature of that complex phenomenon which is here termed "a compound chemical event" as an aggregate or collection of events which concur to produce a specified result, and that result alone, we shall endeavour to arrive at a similar comprehension of the nature of those operations by which events occur, that is to say, of "the causes of events." The fundamental idea through which these operations will be conceived of in this Calculus is given in the following definition.

(1) Definition:—

If in any chemical event the change in the arrangement of the symbols by which the composition of the units of matter before and after the event respectively is symbolized be of such a nature that where in the arrangement before the event the symbol $x$ appears the symbol $a$ appears in the arrangement after that event, and where the symbol $a$ appears in the arrangement before that event the symbol $x$ appears in the arrangement after that event ($x$ and $a$ being symbols of two among the prime factors by which the equation is expressed), so that the two arrangements differ in this respect, and in this respect alone, then that event is said to occur by the "substitution" or "exchange" of $a$ for $x$, and that "substitution" is said to be the "cause" of that event.

It is evident that, consistently with this definition, the same events may arise from more than one "cause;" for it may be true that where, in the arrangement before the event, the symbol $x$ appears the symbol $a$ appears in the arrangement after that event, and that where the symbol $x$ appears in the arrangement after that event the symbol $a$ appears in the arrangement before that event, so that the two arrangements differ in this respect, and in this respect alone; but it may also be true that where in the arrangement before the event the symbol $y$ appears the symbol $b$ appears in the arrangement after that event, and where the symbol $b$ appears in the arrangement before the event the symbol $y$ appears in the arrangement after the event, so that the two arrangements differ in this respect, and in this respect alone. In this case the event may be referred to *either* of *two* "causes," namely, to the substitution of $a$ for $x$ or for $b$ for $y$. These causes represent two alternative hypotheses as to the ways in which the event may have occurred. Similarly one and the same "event" may be referred to several alternative "causes," representing various hypotheses as to the modes of the occurrence of that event.

(2) If, however, the two arrangements so differ that where the symbol $x$ appears in the arrangement before the event $a$ appears in the arrangement after the event, and where $a$ appears in the arrangement before the event $x$ appears in the arrangement after the event, and also differ in the circumstance that where the symbol $y$ appears in the arrangement before the event $b$ appears in the arrangement after the event, and where $b$ appears in the arrangement before the event $y$ appears in the arrangement after the event, then that event is said to occur by two substitutions, namely, by the substitution of $a$ for $x$ and of $b$ for $y$; and these two substitutions are said to be the two "causes" of that event for the production of which they concur; and, similarly, if the arrangements referred to differ in the $n$ circumstances that where $x, y, z\ldots$

appear in the arrangement before that event, $a, b, c \ldots$ appear in the arrangement after that event, and where $a, b, c \ldots$ appear in the arrangement before that event, $x, y, z \ldots$ appear in the arrangement after that event, then the event is said to occur by the $n$ substitutions of $a$ for $x$, $b$ for $y$, $c$ for $z$, and these substitutions are termed the $n$ causes of the event which concur for its production.

The same remark is applicable here as in the case of an event occurring by a single "cause," namely, that as events may occur by a single "cause" in any number of ways, so also may we have events occurring by a set of two or more "causes" in any number of ways, each set of "causes" representing an alternative hypothesis as to the mode of the occurrence of that event.

(3) Definition:—

If an event be regarded as occurring by the substitution of $a$ for $x$, $x$ is termed the symbol of the "variable" in that event, and $a$ is termed a "value" of $x$ (being that for which $x$ is exchanged); if the event be regarded as occurring by the substitutions of $a$ for $x$, $b$ for $y$, $c$ for $z$, $x, y, z$ are termed the symbols of the "variables" in that event, and $a, b, c$ the "values" of those "variables," and so on for any number of substitutions.

(4) Definition:—

A "constant weight" is a weight which, in any specified event or system of events, is *not* exchanged for any other weight.

(5) It is a consequence of the above definitions that the "substitutions" by which events occur will be indicated to us by certain algebraical properties of the equations which express the results of those events. Thus, if an event occurs by the substitution of $a$ for $x$, it is evident, from the definition, that the equation corresponding to the event must vanish when $x$ is put equal to $a$; and if the event occurs by one substitution, and in one way only by such a substitution, so that it can be referred to one cause alone, the equation will vanish under this condition, and under no other similar condition. If there be two alternative "causes" of the event, so that the event may be regarded as occurring either by the substitution of $a$ for $x$, or of $b$ for $y$, the equation will vanish when $x$ is put equal to $a$, and also will vanish when $y$ is put equal to $b$; and if there be $n$ alternative causes of the event, so that the event may occur either by the substitution of $a$ for $x$, of $b$ for $y$, of $c$ for $z, \ldots$ the equation will vanish in $n$ ways, namely, when either $x$ is put equal to $a$, or $y$ is put equal to $b$, or $z$ is put equal to $c, \ldots$ If the event may be referred to two "causes" which concur for its production, so that it occurs by two substitutions, namely, the substitutions of $a$ for $x$ and of $b$ for $y$, the equation will vanish if in that equation we simultaneously put $x$ equal to $a$ and $y$ equal to $b$; and if the event may be referred to $n$ "causes" which concur for its production, so that it occurs by $n$ substitutions of $a$ for $x$, $b$ for $y$, $c$ for $z \ldots$, the equation will vanish if in that equation we simultaneously put $x$ equal to $a$, $y$ equal to $b$, $z$ equal to $c \ldots$ And, similarly, if the equation may not only be referred to $n$ causes, but may be referred in $m$ ways to $n$ causes, there will then be $m$ ways in which the equation will vanish under a similar condition.

It is necessary to notice that every algebraical condition under which a chemical equation may vanish does not necessarily correspond to some one among the "causes" of the event, or throw light upon such causes. For an equation may vanish under algebraical conditions which are chemically uninterpretable and unmeaning. But if an equation vanish under certain specified conditions, such as that $x=a$, when $x$ and $a$ are two of the prime factors of an equation constructed upon the principles before laid down (I. Sec. V., II. Sec. I.), these conditions are capable of interpretation, and inform us of the "real" causes of the event—the term "real" causes being here used to distinguish such causes from "imaginary" causes, which may be defined as substitutions corresponding to algebraical conditions which are uninterpretable. It is quite possible that the consideration of "imaginary" causes may hereafter find its place in the chemical Calculus and lead to true results, but we shall not now consider them.

(6) I shall now proceed to the consideration of certain forms of equations corresponding to the events of which the definition has now been given, and the interpretation of those forms.

I will commence with the equation already referred to (Part II. Sec. I. (4)),

$$\mathrm{A}xy+\mathrm{A}ab=\mathrm{A}ya+\mathrm{A}xb.$$

Now, if we compare the arrangement before the event, namely, $\mathrm{A}xy+\mathrm{A}ab$, with the arrangement after the event, namely, $\mathrm{A}ay+\mathrm{A}xb$, superposing those arrangements the one upon the other, thus—

$$\mathrm{A}xy+\mathrm{A}ab, \quad \ldots\ldots\ldots\ldots \quad \mathrm{I.}$$
$$\mathrm{A}ay+\mathrm{A}xb, \quad \ldots\ldots\ldots\ldots \quad \mathrm{II.}$$

it will be seen that where $x$ appears in the arrangement before the event, $a$ appears in the arrangement after the event, and where $x$ appears in the arrangement after the event, $a$ appears in the arrangement before the event, and the two arrangements differ in this respect, and in this respect alone. Thence, according to the definition, the event symbolized in the above equation may occur by the substitution of $a$ for $x$, and this substitution is a cause of that event.

But, since $\mathrm{A}ay+\mathrm{A}xb=\mathrm{A}xb+\mathrm{A}ay$, we may compare these arrangements from another point of view; for, writing the arrangements after the event as $\mathrm{A}xb+\mathrm{A}ay$, and again, as before, superposing the arrangements the one upon the other, thus—

$$\mathrm{A}xy+\mathrm{A}ab, \quad \ldots\ldots\ldots\ldots \quad \mathrm{I.}$$
$$\mathrm{A}xb+\mathrm{A}ay, \quad \ldots\ldots\ldots\ldots \quad \mathrm{II.}$$

it will be seen that where $y$ appears in the arrangement before the event (I.), $b$ appears in the arrangement after the event (II.), and where $b$ appears in the arrangement before the event (I.), $y$ appears in the arrangement after the event (II.), and the two arrangements differ in this respect, and in this respect alone. Whence, according to the definition, the event may occur by the substitution of $b$ for $y$, which substitution is a real "cause" of that event.

This event therefore is an event which may be referred to either of two "causes," namely, to the substitution of $a$ for $x$ and of $b$ for $y$, which "causes" represent two alternative hypotheses as to the ways of the occurrence of the event.

But, further, not only is it true that the event may be referred to one or the other of these two causes, the occurrence of either of which is sufficient to account for the event, but the event must be so referred as there is no other substitution whatever which will produce the result in question. This may be proved experimentally by altering the arrangement of the letters in every permissible manner, and, as before, superposing the arrangements.

But it is sufficient to observe that there are only two conceivable cases; for where $\mathrm{A}xy$ appears before the event, either $\mathrm{A}ay$ appears after the event or $\mathrm{A}xb$ so appears. In the former case the result is attained, if possible at all, by the substitution of $a$ for $x$; in the latter case, by the substitution of $b$ for $y$.

We might also, in demonstrating the above proposition, have reasoned thus: having proved that the event may occur by the substitution of $a$ for $x$, since in the above equation we may change the places of $x$ and $y$, provided also we change the places of $a$ and $b$, and the equation will be unaffected by this alteration. Hence, whatever is true of $x$ and $a$ is also true of $y$ and $b$; that is to say, whatever assertion may be made in reference to that equation in regard to $x$ and $a$ is also true if in that assertion we substitute $y$ for $x$ and $b$ for $a$.

Writing the above equation in the form $u=0$, we have, from the distributive law (Part I. Sec. II. (6)),

$$u=\mathrm{A}(x-a)(y-b),$$

from which it is apparent that the substitution by which the event occurs, that is to say, the "causes" of the event, are indicated to us by the factors of this equation, and are coincident with those factors; and, precisely as the factors in a numerical equation of this form indicate to us the two sole conditions under which the numerical identity asserted in the equation is possible, namely, the identity of $x$ and $a$, or the identity of $y$ and $b$, so in the case of this chemical equation the factors indicate to us the two sole conditions, or hypotheses, under which the chemical identity expressed in that equation is possible, namely, the substitution of $a$ for $x$, or the substitution of $b$ for $y$. For the only possible way in which the "weights," or matter, symbolized as $\mathrm{A}xy+\mathrm{A}ab$ can have been chemically converted into the "weights," or matter, symbolized as $\mathrm{A}xb+\mathrm{A}ay$ is by the occurrence of one or other of these two "substitutions," either of which affords an adequate and sufficient cause of the metamorphosis.

As the expression $\mathrm{A}(x-a)(y-b)$ is the symbol of the event of the transformation of $\mathrm{A}xy$ and $\mathrm{A}ab$ into $\mathrm{A}ay$ and $\mathrm{A}bx$, so the expression $-\mathrm{A}(x-a)(y-b)$ is the symbol of the transformation of $\mathrm{A}ay$ and $\mathrm{A}bx$ into $\mathrm{A}xy$ and $\mathrm{A}ab$. This event will be termed, in relation to the former event, the "reverse" of that event or the "reverse event;" and while the symbol $\mathrm{A}(x-a)(y-b)$ is interpreted as the symbol of a "substantive" event in which

A is constant, and which occurs by the two substitutions indicated; the symbo $-\mathrm{A}(x-a)(y-b)$ is to be interpreted as the symbol of an event which is the "reverse" of an event in which A is constant, and which occurs by those two substitutions,—that is to say, in the latter case the event is defined by specifying the relation in which that event stands to another event, the conception of which is essential to its comprehension. This interpretation is a consequence of the identity (Part I. Sec. II. (4))

$$-\mathrm{A}(x-a)(y-b)=+[-\mathrm{A}(x-a)(y-b)].$$

Since

$$\mathrm{A}(x-a)(y-b)=-\mathrm{A}(a-x)(y-b),$$

every event of this kind may be considered from these two points of view, namely, either as a "substantive" event or as the reverse of some other event; and wherever in this Calculus the symbol of a chemical event occurs preceded by the negative sign, thus, $-\mathrm{U}$, that symbol is always to be interpreted as though it were written $+(-\mathrm{U})$, namely, as an event which is the reverse of the event U.

In the light of the preceding observations the reader will interpret the following identities, which are demonstrated from the principles established, I. Sec. II. (4) (6),

$$\mathrm{A}(x-a)(y-b)=\mathrm{A}(a-x)(b-y),$$
$$-\mathrm{A}(x-a)(y-b)=\mathrm{A}(a-x)(y-b),$$
$$-[-\mathrm{A}(x-a)(y-b)]=\mathrm{A}(x-a)(y-b).$$

In the previous reasoning the three properties of chemical symbols are utilized which were demonstrated in Part I. Sec. II., namely, that $xy+ab=ab+xy$, that $xy=yx$, and that $\mathrm{A}(x\pm a)=\mathrm{A}x\pm \mathrm{A}a$. If any one of these properties should not belong to such symbols, the reasoning altogether fails.

For the application to a chemical equation of the preceding principles, the equation must be expressed by means of some specified set of prime factors. Now the only expression of this kind which we have as yet had occasion to consider is the expression of chemical equations by means of the prime factors ascertained in Part I., namely, the prime factors $\alpha$, $\chi$, $\xi$, $\theta$, . . ., which are the symbols of those "weights" which are undistributed in the total system of chemical phenomena. The term "simple" or undistributed weight, however, is a purely relative term, having been defined as a "weight" which in some special system of chemical events is not resolved into two or more weights or made up from such weights (Part I. Sec. I. 12). Hence an equation may be expressed, possibly even in numerous ways, by means of different sets of prime factors according to the system of events to which the event under consideration is referred—these prime factors being indeed the symbols of the "simple weights" among which the "substitutions" are conceived of as occurring which are the causes of the events, every such expression necessarily involves an assertion or an hypothesis as to the composition of some one or more among the units of ponderable matter, of which the transformations are considered, which constitutes the base of the symbolic system, analogous to the fundamental hypothesis which is the base of the system constructed in Part I., namely, that the unit of hydrogen is itself a simple weight and expressed by a single letter $\alpha$. This is not the place for the further discussion of this question, which I shall hereafter have occasion to consider in some detail; but I have made the preceding remarks to prevent any misapprehension arising in the mind of the reader in reference to the examples of chemical events given in this and the following Section, in which the equations are expressed by various sets of prime factors. These are not, as a matter of fact, mere arbitrary expressions (although, indeed, such would have sufficiently served the object in view), but I have not entered on the reasons which justify them, as being beside my purpose. I need only further observe that the equations are not to be considered collectively as representing systems of events (although they may be selected from such systems), but as individual equations, where a system is reduced to its simplest form, namely to a single event which from some unexplained reason is represented in the way given.

(7) The following event is an example of this class of phenomena. A unit of chloride of benzoyl and a unit of potassium alcohol are identical with a unit of chloride of potassium and a unit of benzoic ether,

$$\alpha^3\varkappa^7\xi\chi+\alpha^3\varkappa^2\xi\mu=\alpha\chi\mu+\alpha^5\varkappa^9\xi^2.$$

This equation may be expressed by means of the prime factors $\alpha$, $\chi$, $\mu$, $(\alpha^2\varkappa^7\xi)$, $(\alpha^2\varkappa^2\xi)$, and thus written

$$\alpha(\alpha^2\varkappa^7\xi)\chi+\alpha(\alpha^2\varkappa^2\xi)\mu=\alpha\chi\mu+\alpha(\alpha^2\varkappa^7\xi)(\alpha^2\varkappa^2\xi),$$

whence

$$\alpha(\alpha^2\varkappa^7\xi-\mu)(\chi-\alpha^2\varkappa^2\xi)=0.$$

By which equation we are informed that $\alpha$ in this event is "constant" and that the event occurs in one of two ways, namely, either by the exchange of the "weight" $\alpha^2\varkappa^7\xi$ for $\mu$ or of $\chi$ for $\alpha^2\varkappa^2\xi$. It is to be borne in mind that these hypotheses are purely relative to this equation alone, and involve the assumption that in this event the "weights" $\alpha$, $\alpha^2\varkappa^7\xi$, $\alpha^2\varkappa^2\xi$, $\mu$, $\chi$ are not distributed. If this restriction be removed, other views may be taken of the nature of this occurrence.

(8) It is desirable to notice certain special forms of this fundamental phenomenon. If $a=b$ so that the "values" of $y$ and $x$ become identical, the equation becomes

$$\mathrm{A}(x-a)(y-a)=0,$$

and

$$\mathrm{A}xy+\mathrm{A}a^2=\mathrm{A}ay+\mathrm{A}ax.$$

Such an event, for example, is the following:—

Example:—A unit of acetate of chlorine and a unit of iodine are identical with a unit of acetate of iodine and a unit of the chloride of iodine; thus

$$\alpha^2\varkappa^2\xi^2\chi+\alpha\omega^2=\alpha^2\varkappa^2\xi^2\omega+\alpha\omega\chi,$$

which may be written thus

$$\alpha(\alpha\varkappa^2\xi^2)\chi+a\omega^2=\alpha(\alpha\varkappa^2\xi^2)\omega\chi+\alpha\omega\chi,$$

whence

$$\alpha(\alpha\varkappa^2\xi^2-\omega)(\chi-\omega)=0.$$

In this event, therefore, $\alpha$ is necessarily to be regarded as "constant," and the event is to be referred to one of two "causes," namely, the exchange of $\alpha\varkappa^2\xi^2$ for $\omega$ or of $\chi$ for $\omega$.

If also $x=y$, the equation becomes

$$A(x-a)^2=0,$$

the two causes of the event being identical, and

$$Ax^2+Aa^2=2Aax.$$

Such an event is the following:—

Example:—A unit of chlorine and a unit of iodine are identical with two units of chloride of iodine; thus

$$\alpha\chi^2+a\omega^2=2a\chi\omega,$$

whence

$$\alpha(\chi-\omega)^2=0.$$

In this event $\alpha$ is "constant;" the event occurs by the substitution of $\omega$ for $\chi$, and occurs in two ways by that substitution, as is evident on inspecting the equation.

(9) If in the above equation $a=1$, so that the equation becomes

$$A(x-1)(y-b)=0,$$

one of the "causes" of the event is the substitution of 1 for $x$. Now the symbol 1, being interpreted as the symbol of the "weight" or matter which occupies an empty unit of space, is the symbol of "no weight;" and this "cause," therefore, is the substitution of "no weight" for the weight which results from the performance of the operation $x$. A substitution of this kind will be termed "a transference." Thus we should say that the above event occurred either by the "substitution" of $b$ for $y$, or by the "transference" of $x$.

In introducing this term it is necessary to guard against the supposition that "a transference" is here regarded as a new and peculiar phenomenon distinct from a "substitution." This is not the case; every "transference" is a "substitution," although every substitution is not a "transference." This distinction is, however, really inherent in the received theory of "substitution" or "double decomposition," as explained even by the most competent chemists. Thus KEKULÉ indicates as a blot on the theory of "double decomposition," that this theory is inapplicable to cases of "direct addition," or applicable only with the greatest violence "nicht (oder nur höchst gezwungen) anwendbar" [KEKULÉ, vol. i. p. 142]. In this Calculus the distinction is not abolished, but simply does not appear at all, every chemical event being necessarily referred in it (without any forcing) to the operation of one and the same law of "substitution," a generalization due to the introduction of the chemical symbol 1. The meaning of a remark made in the first part of this Calculus (I. Sec. III. (3)) and there left unexplained, to the effect that the matter of an empty unit of space, although not an object to be presented to the imagination, must nevertheless, if we would reason correctly, be "treated as a reality in the order of ideas," will now be apparent.

Example:—The following event is of the above form,

$$\alpha^3\kappa^2\omega+\alpha^2\nu=\alpha\omega+\alpha^4\kappa^2\nu,$$

which equation may be written thus

$$\alpha(\omega-\alpha\nu)(\alpha^2\kappa^2-1)=0.$$

The event here symbolized is the transformation of a unit of iodide of ethyl and a unit of ammonia into a unit of hydriodic acid and a unit of ethylamine, which event may be referred to one of two causes, namely, either to the substitution of $\alpha\nu$ for $\omega$, or to the transference of $\alpha^2\kappa^2$, as is seen on inspecting the above equation.

Example:—Again,

$$\alpha\xi^2+\alpha=2\alpha\xi,$$

or

$$\alpha(\xi-1)^2=0.$$

This equation expresses the event of the transformation of a unit of hydrogen and a unit of binoxide of hydrogen into two units of water.

This event occurs by "the transference" of $\xi$, and in two ways by that "transference."

Example:—Again take the equation

$$\alpha^5\kappa^4\xi+\alpha\xi=2\alpha^3\kappa^2\xi,$$

as expressed by the prime factors $\alpha$, $\xi$, $(\alpha^2\kappa^2)$,

$$\alpha\xi(\alpha^2\kappa^2)^2+\alpha\xi=2\alpha\xi(\alpha^2\kappa^2),$$

or

$$\alpha\xi(\alpha^2\kappa^2-1)^2=0,$$

which symbolizes the event of the transformation of a unit of ether and a unit of water into two units of alcohol.

In this event $\alpha\xi$ (the unit of water) is necessarily to be regarded as constant. The event occurs by "the transference" of $\alpha^2\kappa^2$, and in two ways by that "transference.

If in the equation $A(x-a)(y-b)=0$, $A=1$, so that

$$(x-a)(y-b)=0,$$

"no weight" is constant, the event being fully defined when we say that the event occurs by the substitution of $a$ for $x$ or of $b$ for $y$.

Example:—Such an event, for example, is the transformation of a unit of hydrogen and a unit of sulphide of mercury into a unit of mercury and a unit of sulphide of hydrogen; thus

$$\delta\theta+\alpha=\alpha\theta+\delta,$$

$$(\delta-\alpha)(\theta-1)=0.$$

In which event "no weight" is constant, and the event occurs either by the "substitution" of $\alpha$ for $\delta$ or by the transference of $\theta$.

Example:—Or, again, the transformation of a unit of binoxide of hydrogen into a unit of hydrogen and a unit of oxygen, which may be thus expressed,

$$(\alpha-1)((\xi^2)-1)=0.$$

In which event, again, "no weight" is constant, and the event occurs either by the transference of $\alpha$ or the transference of $\xi^2$, as is apparent from the equation

$$1+\alpha(\xi^2)=\alpha+(\xi^2).$$

(10) The consideration of an event, not as a "substantive" event, but as "the reverse" of some other event, is a principle of very real utility, both as regards our appreciation of the real "causes" of events, and also as regards our appreciation of the analogies and relations of events.

Take, for example, the event of the transformation of a unit of binoxide of hydrogen and a unit of sulphurous acid into a unit of sulphuric acid, the result of which is given in the equation

$$\alpha\xi^2+\theta\xi^2=1+\alpha\theta\xi^4.$$

Now this equation may be expressed by the prime factors $(\alpha\xi^2)$ and $(\theta\xi^2)$ and written thus,

$$(\alpha\xi^2)+(\theta\xi^2)=1+(\alpha\xi^2)(\theta\xi^2).$$

The symbol of this occurrence, regarded as a substantive event, is

$$(1-\alpha\xi^2)(\theta\xi^2-1)=0,$$

which compels us to interpret the factor $1-(\alpha\xi^2)$, in which case it would be necessary to introduce the symbol 1 into the equation in an explicit form. But these difficulties disappear when we write the equation thus,

$$+[-(\alpha\xi^2-1)(\theta\xi^2-1)]=0.$$

In this case we do not define the above event as a substantive event; but we simply say that it is the reverse of another event, the nature of which we can readily comprehend, the two alternative causes of which are the transference of $(\alpha\xi^2)$ and the transference of $(\theta\xi^2)$.

Again, the equation

$$\alpha+\alpha\chi^2=2\alpha\chi,$$

which expresses the formation of hydrochloric acid, may be written thus,

$$\alpha(\chi-1)^2=0,$$

being an event in which $\alpha$ is constant, and which occurs in two ways by the transference of $\chi$.

Now the resolution of hydrochloric acid into its elements is thus expressed,

$$2\alpha\chi=\alpha+\alpha\chi^2,$$

the symbol of which, regarded as a substantive event, is

$$\alpha(1-\chi)(\chi-1)=0.$$

Instead of attempting to interpret the factor $(1-\chi)$ we may write this event thus,

$$+[-\alpha(\chi-1)^2]=0;$$

in which case we do not attempt to explain the "causes" of the event itself, but instead of so doing, we say that the event is the reverse of an event of which both causes are intelligible to us.

The events which we have now considered are completely defined by their symbolical expression. They are events which may be referred to two alternative "causes," and to two such "causes" alone; that is to say, they are events which necessarily occur by one or the other of two specified substitutions, and by this property are, as a class, separated from all other phenomena. This great class of events is, however, only one, and that the very simplest, instance of an indefinite number of such systems to the conception and consideration of which we are brought by the development of the methods of this Calculus, and to the explanation of which I shall now proceed. Of these systems, also, I shall lay before the reader a sufficient number of examples to invest these conceptions with reality, and to satisfy him that we are not dealing simply with algebraical forms, but with algebraical forms corresponding to real occurrences. It will not, however, be necessary to dwell at equal length upon this portion of the subject; for the reader who has accompanied me so far will readily appreciate what I have now to submit to him.

(11) If an equation be expressed as the continued product of three factors each of the form $x-a$, so that

$$A(x-a)(y-b)(z-c)=0,$$

the event may be conceived of as occurring by one substitution and, as thus occurring, in three different ways, namely, by the substitution of $a$ for $x$ and of $b$ for $y$ and of $c$ for $z$, which three "substitutions" are to be regarded as three alternative "causes" of that event, to the one or the other of which that event must necessarily be referred. For the result of this event is expressed in the equation

$$Axyz+Aabz+Aayc+Axbc=Axyc+Aabc+Aayz+Axbz;$$

and if, proceeding as before, we institute a comparison between the arrangement before the event and the arrangement after the event, with the view of ascertaining experimentally the substitutions which are the causes of the event, superposing the one arrangement upon the other, thus—

$$\mathrm{A}xyz+\mathrm{A}abz+\mathrm{A}ayc+\mathrm{A}xbc, \quad \ldots\ldots\ldots \quad \mathrm{I.}$$

$$\mathrm{A}xyc+\mathrm{A}abc+\mathrm{A}ayz+\mathrm{A}xbz, \quad \ldots\ldots\ldots \quad \mathrm{II.}$$

it is apparent that where $z$ appears in the arrangement (I.) before the event $c$ appears in the arrangement (II.) after the event, and where $c$ appears in the arrangement (I.) before the event $z$ appears in the arrangement (II.) after the event, and the arrangements differ in that respect, and in that respect alone. That is to say, according to the definition, Section III. (1), the event occurs by the substitution of $c$ for $z$. But since

$$\mathrm{A}(x-a)(y-b)(z-c)=\mathrm{A}(x-a)(z-c)(y-b),$$

the above reasoning in regard to $z$ and $c$ is equally correct with regard to $y$ and $b$, that is to say, the event also occurs by the "substitution" of $b$ for $y$; and similarly the event also occurs by the "substitution" of $a$ for $x$; and there are no other two letters in regard to which a similar assertion can be made.

(12) Every actual example of such an event in which $x$, $y$, $z$ and $a$, $b$, $c$ are all different involves the construction of no less than eight different chemical substances connected by the relation of identity expressed in the above equation. The problem of the realization of such an event is greatly simplified where two or more among the letters $x$, $y$, $z$ or $a$, $b$, $c$ become identical. The simplest case, from this point of view (the case, that is, which involves the construction of the smallest number of different substances), is that in which $x=y=z$, and also $a=b=c$. The equation then becomes

$$\mathrm{A}(x-a)^3=0,$$

and

$$\mathrm{A}x^3+3\mathrm{A}a^2x=3\mathrm{A}ax^2+\mathrm{A}a^3.$$

Example:—A unit of chloroform and three units of chloride of methyl are identical with three units of mono-methyl chloride and a unit of marsh-gas; thus

$$(\alpha^2\kappa)\chi^3+3(\alpha^2\kappa)\chi=3(\alpha^2\kappa)\chi^2+(\alpha^2\kappa),$$

whence

$$\alpha^2\kappa(\chi-1)^3=0.$$

The event therefore is fully defined by saying that the above phenomenon is an event in which $\alpha^2\kappa$ (the symbol of the unit of marsh-gas) is "constant," and which occurs by the transference of $\chi$, and occurs by that transference in three ways.

Example:—A unit of the trichlorhydrine of glycerine and three units of monochlorhydrine are identical with a unit of glycerine and three units of dichlorhydrine; thus

$$\alpha^4\kappa^3\chi^3+3\alpha^4\kappa^3\xi^2\chi=\alpha^4\kappa^3\xi^3+3\alpha^4\kappa^3\xi\chi^2,$$

whence

$$\alpha^4\kappa^3(\chi-\xi)^3=0.$$

In this event $\alpha^4\kappa^3$, the unit of the (so called) hydride of propyl, is constant. The event occurs by the substitution of $\xi$ for $\chi$, and by that substitution in three ways.

Example:—A unit of glycerine and three units of diacetine are identical with three units of monacetine and a unit of triacetine,

$$\alpha^4\varkappa^3\xi^3+3\alpha^6\varkappa^7\xi^5=3\alpha^5\varkappa^5\xi^4+\alpha^7\varkappa^9\xi^6,$$

which equation may be written thus,

$$\alpha^4\varkappa^3\xi^3+3\alpha^4\varkappa^3\xi^3(\alpha\varkappa^2\xi)^2=3\alpha^4\varkappa^3\xi^3(\alpha\varkappa^2\xi)+\alpha^4\varkappa^3\xi^3(\alpha\varkappa^2\xi)^3,$$

whence

$$-\alpha^4\varkappa^3\xi^3(\alpha\varkappa^2\xi-1)^3.$$

That is to say, this event is the reverse of an event in which $\alpha^4\varkappa^3\xi^3$ (the unit of glycerine) is constant, and which occurs by the transference of $\alpha\varkappa^2\xi$, and in three ways by that transference.

If in the above equation $x=y$ and $a=b=c$, the equation becomes

$$A(x-a)^2(z-a)=0,$$

and

$$Ax^2z+2Aa^2x+Aa^2z=Aax^2+2Aaxz+Aa^3.$$

Example:—A unit of ethyldiacetamide and two units of acetamide and a unit of ethylamine are identical with a unit of diacetamide and two units of diethylmonacetamide and a unit of ammonia [ref. KEKULÉ, vol. i. p. 575]; thus

$$\alpha^6\varkappa^6\xi^2\nu+2\alpha^3\varkappa^2\xi\nu+\alpha^4\varkappa^2\nu=\alpha^4\varkappa^4\xi^2\nu+2\alpha^6\varkappa^4\xi\nu+\alpha^2\nu,$$

which equation may be put in the following form,

$$\alpha^2\nu(\alpha\varkappa^2\xi)^2(\alpha^2\varkappa^2)+2\alpha^2\nu(\alpha\varkappa^2\xi)+\alpha^2\nu(\alpha^2\varkappa^2)=\alpha^2\nu(\alpha\varkappa^2\xi)^2+2\alpha^2\nu(\alpha\varkappa^2\xi)(\alpha^2\varkappa^2)^2+\alpha^2\nu,$$

whence

$$\alpha^2\nu(\alpha\varkappa^2\xi-1)^2(\alpha^2\varkappa^2-1)=0.$$

In this event $\alpha^2\nu$ (the unit of ammonia) is constant, and the event occurs by the transference of $\alpha^2\varkappa^2$ and by the transference of $\alpha\varkappa^2\xi$, and by the latter transference in two ways.

Example:—A unit of diethylamylamine, a unit of amylamine, and two units of ethylamine are identical with a unit of ammonia, a unit of diethylamine, and two units of ethylamylamine,

$$\alpha^{11}\varkappa^9\nu+\alpha^7\varkappa^5\nu+2\alpha^4\varkappa^2\nu=\alpha^2\nu+\alpha^6\varkappa^4\nu+2\alpha^9\varkappa^7\nu.$$

This equation may be written thus,

$$\alpha^2\nu(\alpha^2\varkappa^2)^2(\alpha^5\varkappa^5)+\alpha^2\nu(\alpha^5\varkappa^5)+2\alpha^2\nu(\alpha^2\varkappa^2)=\alpha^2\nu+\alpha^2\nu(\alpha^2\varkappa^2)^2+2\alpha^2\nu(\alpha^2\varkappa^2)(\alpha^5\varkappa^5),$$

whence

$$\alpha^2\nu(\alpha^2\varkappa^2-1)^2(\alpha^5\varkappa^5-1)=0.$$

In this event $\alpha^2\nu$ (the unit of ammonia) is constant. The event occurs either by the transference of $\alpha^5\varkappa^5$ or by the transference of $\alpha^2\varkappa^2$, and by the latter transference in one of two ways.

If $a=b=c$, the equation becomes

$$A(x-a)(y-a)(z-a)=0,$$

and

$$Axyz+Aa^2x+Aa^2y+Aa^2z=Aaxy+Aaxz+Aayz+Aa^3.$$

Example:—A unit of methylethylamylamine and a unit of methylamine and a unit of ethylamine and a unit of amylamine are identical with a unit of methylethylamine and a unit of methylamylamine and a unit of ethylamylamine and a unit of ammonia; thus

$$\alpha^{10}\varkappa^8\nu+\alpha^3\varkappa\nu+\alpha^4\varkappa^2\nu+\alpha^7\varkappa^5\nu=\alpha^5\varkappa^3\nu+\alpha^8\varkappa^6\nu+\alpha^9\varkappa^7\nu+\alpha^2\nu,$$

which equation may be written thus,

$$(\alpha^2\nu)(\alpha\varkappa)(\alpha^2\varkappa^2)(\alpha^5\varkappa^5)+(\alpha^2\nu)(\alpha\varkappa)+(\alpha^2\nu)(\alpha^2\varkappa^2)+(\alpha^2\nu)(\alpha^5\varkappa^5)=(\alpha^2\nu)(\alpha\varkappa)(\alpha^2\varkappa^2)$$
$$+(\alpha^2\nu)(\alpha\varkappa)(\alpha^5\varkappa^5)+(\alpha^2\nu)(\alpha^2\varkappa^2)(\alpha^5\varkappa^5)+(\alpha^2\nu),$$

whence

$$\alpha^2\nu(\alpha\varkappa-1)(\alpha^2\varkappa^2-1)(\alpha^5\varkappa^5-1)=0\,;$$

$\alpha^2\nu$ (the unit of ammonia) is constant, and the event occurs by the transference of $\alpha\varkappa$, and by the transference of $\alpha^2\varkappa^2$, and by the transference of $\alpha^5\varkappa^5$.

If $a=b$ the equation becomes

$$A(x-a)(y-a)(z-c)=0,$$

and

$$Axyz+Aacx+Aacy+Aa^2z=Acxy+Aaxz+Aayz+Aa^2c.$$

Example:—A unit of the acetobromochlorhydrine of glycerine and a unit of monacetine and a unit of the monobromohydrine of glycerine and a unit of the monochlorhydrine of glycerine are identical with a unit of glycerine and a unit of the bromochlorhydrine of glycerine and a unit of the acetobromohydrine of glycerine and a unit of the acetochlorhydrine of glycerine, whence

$$\alpha^5\varkappa^5\xi^2\beta\chi+\alpha^5\varkappa^5\xi^4+\alpha^4\varkappa^3\xi^2\beta+\alpha^4\varkappa^3\xi^2\chi=\alpha^4\varkappa^3\xi^3+\alpha^4\varkappa^3\xi\beta+\alpha^5\varkappa^5\xi^3\beta+\alpha^5\varkappa^5\xi^3\chi.$$

This equation may be written thus,

$$(\alpha^4\varkappa^3\xi)(\alpha\varkappa^2\xi)\beta\chi+(\alpha^4\varkappa^3\xi)(\alpha\varkappa^2\xi)\xi^2+(\alpha^4\varkappa^3\xi)\beta\xi+(\alpha^4\varkappa^3\xi)\xi\chi=(\alpha^4\varkappa^3\xi)\xi^2$$
$$+(\alpha^4\varkappa^3\xi)\beta\chi+(\alpha^4\varkappa^3\xi)(\alpha\varkappa^2\xi)\beta\xi+(\alpha^4\varkappa^3\xi)(\alpha\varkappa^2\xi)\xi\chi,$$

whence

$$\alpha^4\varkappa^3\xi(\beta-\xi)(\chi-\xi)(\alpha\varkappa^2\xi-1)=0.$$

We are thus informed that in this event $\alpha^4\varkappa^3\xi$ is constant, and that there are three causes of the event, namely, the "transference" of $\alpha\varkappa^2\xi$, the substitution of $\xi$ for $\chi$, and the substitution of $\xi$ for $\beta$.

(13) By reasoning analogous to that previously employed, it may be readily demonstrated that if a chemical equation be expressed by four factors of the form $x-a$, so that

$$A(x-a)(y-b)(z-c)(v-d)=0,$$

the event symbolized in that equation may occur by one substitution, and may thus occur in four different ways; that is to say, there are four alternative "causes" of that event, to one or the other of which the event must necessarily be referred, namely, the substitution of $a$ for $x$, of $b$ for $y$, of $c$ for $z$, and of $d$ for $v$. The same principle may be extended to any number of factors; and generally, if a chemical equation be expressed by $n$ such factors, so that

$$A(x-a)(y-b)(z-c)(v-d)(w-e)\ldots\ldots=0,$$

the event symbolized in that equation may occur by one substitution, and may thus occur in $n$ ways; that is to say, there are $n$ alternative "causes" of that event to one or other of which the event must necessarily be referred, namely, the $n$ substitutions indicated by the $n$ factors of that equation.

As the complexity of chemical phenomena increases so do these phenomena, regarded as actual events, become more rare, and it is more and more difficult to give real examples of them. This arises mainly perhaps from the difficulty inherent in the way of their experimental realization, an event expressed even by four different factors requiring the construction of no less than sixteen different substances connected by a certain definite relation, but also from the circumstance that the efforts of chemists have never been directed to the realization of such phenomena as a definite problem. We have the strongest evidence in favour of the existence of these relations, but we cannot exhibit them in a complete form. Such events are, if we may so say, fragmentary and imperfect, like buildings in process of construction, some near to completion, others of which the plan and outline are visible to the eye of the architect alone.

The following events are examples of this class:—

Example:—A unit of tetrachloride of carbon together with six units of monomethyl chloride and a unit of marsh-gas are identical with four units of chloroform and four units of chloride of methyl; thus

$$(\alpha^2\varkappa)\chi^4+6(\alpha^2\varkappa)\chi^2+(\alpha^2\varkappa)=4(\alpha^2\varkappa)\chi^3+4(\alpha^2\varkappa)\chi,$$

whence

$$(\alpha^2\varkappa)(\chi-1)^4=0.$$

In this event $\alpha^2\varkappa$, the symbol of the unit of marsh-gas, is constant. The event occurs by the transference of $\chi$, and in four ways by that transference.

Example:—A unit of tetrachloride of tin, a unit of stannic diethylchloride, four units of stannic ethylmethylchloride [unknown], a unit of stannic dimethylchloride, and a unit of stannic diethyldimethyl are identical with two units of stannic methyltrichloride, two units of stannic methylchloridediethyl [unknown], two units of stannic ethylchloridedimethyl, and two units of stannic ethyltrichloride*, whence

$$\alpha^2\varkappa'\chi^4+\alpha^6\varkappa'\varkappa^4\chi^2+4\alpha^5\varkappa'\varkappa^3\chi^2+\alpha^4\varkappa'\varkappa^2\chi^2+\alpha^8\varkappa'\varkappa^6=2\alpha^3\varkappa'\varkappa\chi^3+2\alpha^7\varkappa'\varkappa^5\chi+2\alpha^6\varkappa'\varkappa^4\chi+2\alpha^4\varkappa'\varkappa^2\chi^3.$$

This equation may be written thus,

$$(\alpha^2\varkappa')\chi^4+(\alpha^2\varkappa')(\alpha^2\varkappa^2)^2\chi^2+4(\alpha^2\varkappa')(\alpha\varkappa)(\alpha^2\varkappa^2)\chi^2+(\alpha^2\varkappa')(\alpha\varkappa)^2\chi^2+(\alpha^2\varkappa')(\alpha\varkappa)^2(\alpha^2\varkappa^2)^2$$
$$=2(\alpha^2\varkappa')(\alpha\varkappa)\chi^3+2(\alpha^2\varkappa')(\alpha\varkappa)(\alpha^2\varkappa^2)^2\chi+2(\alpha^2\varkappa')(\alpha\varkappa)^2(\alpha^2\varkappa^2)\chi+2(\alpha^2\varkappa')(\alpha^2\varkappa^2)\chi^3,$$

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* *Vide* KEKULÉ, vol. i. 782, p. 505.

whence

$$(\alpha^2\kappa)(\chi-\alpha\kappa)^2(\chi-\alpha^2\kappa^2)^2=0.$$

This event, therefore, is to be considered as occurring by the substitution of $\alpha\kappa$ for $\chi$ and by the substitution of $\alpha^2\kappa^2$ for $\chi$, and in two ways by each of these substitutions respectively.

### SECTION IV.

We thus arrive at the interpretation of the chemical equation

$$A(x-a)(y-b)(z-c)\ldots=0,$$

considered as the symbol of a "simple chemical event." From the consideration of such an equation we are led to refer the event thus symbolized to certain specified substitutions (indicated to us by the factors of the equation) as the causes of that event, by the enumeration of which causes, together with the constant A, the event is defined, there being only one possible event which will satisfy the conditions specified in the equation. Such equations constitute a class apart. The explanation of a chemical event consists in referring the event to the causes which concur to produce it. Therefore it is only those events which can be thus expressed which can, in any proper sense, be said to be explained, simple events, and such events admit of no further explanation of this kind.

Now a chemical equation may not be of the above form, an equation to a "simple event," but yet may be expressed by means of rational factors. In this case the event indicated in the equation may be immediately resolved into some number of simple events occurring by one or more specified substitutions, of which that event is the aggregate. Such aggregates were the first actually discovered by me; and although in the next section the whole question of the analysis of aggregates will be discussed in a far more complete manner, I shall yet invite the reader, with a view to a clear comprehension of the subject, to follow the course actually pursued, and to consider a few of the simpler problems which such equations present to us.

### *Examples.*

(1) A unit of chlorine and two units of hydriodic acid are identical with a unit of iodine and two units of hydrochloric acid, and therefore may be transformed into these units.

The equation to this event is

$$\alpha\chi^2+2\alpha\omega=2\alpha\chi+\alpha\omega^2,$$

which equation may be thus expressed,

$$\alpha(\chi+\omega-2)(\chi-\omega)=0.$$

Now we cannot interpret the equation in this form; for although, it is true, we know what is meant by the factor $(\chi-\omega)$ and can perform the substitution which it directs us to perform, the factor $(\chi+\omega-2)$, which should express the alternative mode of the occurrence of the event, conveys no meaning to us. We can, however, easily convert the equation into an interpretable form by resolving this factor. If this be effected the equation to the event becomes

$$\alpha(\chi-1)(\chi-\omega)+\alpha(\omega-1)(\chi-\omega)=0.$$

From the principles demonstrated in Section I. (4) each of the terms of this equation may be equated to zero, and we are thus informed that this event is an aggregate of the two simple events,

$$\alpha(\chi-1)(\chi-\omega)=0,$$
$$\alpha(\omega-1)(\chi-\omega)=0.$$

Since these equations have a factor in common $(\chi-\omega)$, the events expressed in them may be referred to a common cause, namely, the substitution of $\omega$ for $\chi$, and may be regarded as members of a system of two events (in which $\alpha$ is constant) occurring by this substitution. At the same time we have in each case an alternative cause of the event, namely, in the former of the two events the transference of $\chi$, in the latter the transference of $\omega$, the original event regarded in its result being

$$\alpha\chi^2+2\alpha\omega=2\alpha\chi+\alpha\omega^2,$$

and the constituents of that event similarly regarded being

$$\alpha\chi^2+\alpha\omega=\alpha\chi+\alpha\omega\chi,$$
$$\alpha\omega\chi+\alpha\omega=\alpha\omega^2+\alpha\chi.$$

The synthesis here indicated has been actually effected, the unit $\alpha\omega\chi$ being no other than the unit of the chloride of iodine, which is formed, together with a unit of hydrochloric acid, by the action of a unit of chlorine on a unit of hydriodic acid according to the former equation, and a unit of which, together with a unit of hydriodic acid, is resolved into a unit of iodine and a unit of hydrochloric acid according to the latter equation.

(2) A unit of chlorosulphuric acid and two units of water are identical with a unit of sulphuric acid and two units of hydrochloric acid, thus

$$\alpha\theta\xi^2\chi^2+2\alpha\xi=\alpha\theta\xi^4+2\alpha\chi.$$

This equation may be thus expressed,

$$\alpha(\theta\xi^2\chi+\theta\xi^3-2)(\chi-\xi)=0.$$

Here, again, we cannot interpret one of the factors of the equation, namely $(\theta\xi^2\chi+\theta\xi^3-2)$; but proceeding as before we may resolve this factor, and bring the equation into an interpretable form, thus

$$\alpha(\theta\xi^2\chi-1)(\chi-\xi)+\alpha(\theta\xi^3-1)(\chi-\xi)=0.$$

Equating as before each term of this equation to zero, we are informed that the event is an aggregate of the two events

$$\alpha(\theta\xi^2\chi-1)(\chi-\xi)=0,$$
$$\alpha(\theta\xi^3-1)(\chi-\xi)=0.$$

In this case, again, the two events may be referred to a common cause, namely the substitution of $\xi$ for $\chi$. But in each case an alternative is presented to us, namely, in the former of the two events the transference of $(\theta\xi^2\chi)$, regarded as the symbol of a simple weight, and one of the prime factors of the equation, and in the latter event the transference of $(\theta\xi^3)$ similarly regarded.

The event

$$\alpha\theta\xi^2\chi^2+2\alpha\xi=\alpha\theta\xi^4+2\alpha\chi$$

is thus resolved into the constituents

$$\alpha\theta\xi^2\chi^2+\alpha\xi=\alpha\theta\xi^3\chi+\alpha\chi,$$
$$\alpha\theta\xi^3\chi+\alpha\xi=\alpha\theta\xi^4+\alpha\chi,$$

both of which events may be considered to occur by the substitution of $\xi$ for $\chi$.

The synthesis of this phenomenon here suggested to us has actually been realized—the unit of matter, $\alpha\theta\xi^3\chi$, which is thus brought under our notice being the unit of the substance termed hydrochlorosulphurous acid, which is formed, together with other products, by the action of pentachloride of phosphorus on sulphuric acid, and is to be recognized as identical with the substance the unit of which appears in these equations by the property which it possesses of being resolved, together with a unit of water, into a unit of hydrochloric acid and a unit of sulphuric acid.

(3) A unit of bromide of ethylene and two units of water are resoluble into two units of hydrobromic acid and a unit of glycol, thus

$$\alpha^3\varkappa^2\beta^2+2\alpha\xi=2\alpha\beta+\alpha^3\varkappa^2\xi^2.$$

This equation may be thus expressed

$$\alpha(\alpha^2\varkappa^2\beta+\alpha^2\varkappa^2\xi-2)(\beta-\xi)=0,$$

whence

$$\alpha(\alpha^2\varkappa^2\beta-1)(\beta-\xi)+\alpha(\alpha^2\varkappa^2\xi-1)(\beta-\xi)=0,$$

the constituents of the event being

$$\alpha(\alpha^2\varkappa^2\beta-1)(\beta-\xi)=0,$$
$$\alpha(\alpha^2\varkappa^2\xi-1)(\beta-\xi)=0.$$

In these events $\alpha$ is constant; the two events may be referred to a common cause, namely the substitution of $\xi$ for $\beta$. The results are

$$\alpha^3\varkappa^2\beta^2+\alpha\xi=\alpha\beta+\alpha^3\varkappa^2\beta\xi,$$
$$\alpha^3\varkappa^2\beta\xi+\alpha\xi=\alpha\beta+\alpha^3\varkappa^2\xi^2.$$

These events have been realized, the unit $\alpha^3\varkappa^2\beta\xi$, intermediate in composition between glycol and bromide of ethylene, being the monobromohydrin of glycol.

(4) A unit of diacetic glycol and two units of water are identical with two units of acetic acid and a unit of glycol, thus

$$\alpha^5\varkappa^6\xi^4+2\alpha\xi=2\alpha^2\varkappa^2\xi^2+\alpha^3\varkappa^2\xi^2.$$

This equation may be thus expressed,

$$\alpha\xi(\alpha^2\varkappa^2\xi+\alpha^3\varkappa^4\xi^2-2)(\alpha\varkappa^2\xi-1)=0,$$

whence

$$\alpha\xi(\alpha^2\varkappa^2\xi-1)(\alpha\varkappa^2\xi-1)+\alpha\xi(\alpha^3\varkappa^4\xi^2-1)(\alpha\varkappa^2\xi-1)=0,$$

the constituents of the event being

$$\alpha\xi(\alpha^2\varkappa^2\xi-1)(\alpha\varkappa^2\xi-1)=0,$$
$$\alpha\xi(\alpha^3\varkappa^4\xi^2-1)(\alpha\varkappa^2\xi-1)=0.$$

In these events $\alpha\xi$ is constant; the two events may be referred to a common cause, namely the transference of $\alpha\varkappa^2\xi$. The resulting identities are

$$\alpha^4\varkappa^4\xi^3+\alpha\xi=\alpha^3\varkappa^2\xi^2+\alpha^2\varkappa^2\xi^2,$$
$$\alpha^5\varkappa^6\xi^4+\alpha\xi=\alpha^4\varkappa^4\xi^3+\alpha^2\varkappa^2\xi^2.$$

These phenomena have been realized, the unit $\alpha^4\varkappa^4\xi^3$, which together with a unit of acetic acid is formed by the decomposition of a unit of diacetic glycol and a unit of water, is the unit of monacetic glycol, which has been prepared by the simultaneous decomposition of a unit of bromide of ethylene, a unit of acetate of potash, and a unit of water.

(5) A unit of suberic acid is identical with a unit of the hydride of hexyl and two units of carbonic acid; this relation is given in the equation

$$2+\alpha^7\varkappa^8\xi^4=\alpha^7\varkappa^6+2\varkappa\xi^2.$$

This equation may be thus expressed

$$(\alpha^7\varkappa^7\xi^2+\alpha^7\varkappa^6-2)(\varkappa\xi^2-1)=0,$$

whence

$$(\alpha^7\varkappa^7\xi^2-1)(\varkappa\xi^2-1)+(\alpha^7\varkappa^6-1)(\varkappa\xi^2-1)=0,$$

the constituents of the event being

$$(\alpha^7\varkappa^7\xi^2-1)(\varkappa\xi^2-1)=0,$$
$$(\alpha^7\varkappa^6-1)(\varkappa\xi^2-1)=0,$$

which two events may be referred to a common cause, namely the transference of $\varkappa\xi^2$, the results being

$$\alpha^7\varkappa^8\xi^4+1=\alpha^7\varkappa^7\xi^2+\varkappa\xi^2,$$
$$\alpha^7\varkappa^7\xi^2+1=\alpha^7\varkappa^6+\varkappa\xi^2.$$

The unit $\alpha^7\varkappa^7\xi^2$, which appears in these events, but not in the original event, is the unit of œnanthylic acid, the existence of which unit and the properties by which it may be recognized are here pointed out.

(6) A unit of terchloride of phosphorus and three units of water are resoluble into a unit of hydrated phosphorous acid and three units of hydrochloric acid, thus

$$\alpha^2\varphi\chi^3+3\alpha\xi=3\alpha\chi+\alpha^2\varphi\xi^3.$$

This equation vanishes when $\chi=\xi$, and may be thus expressed

$$\alpha(\alpha\phi\chi^2+\alpha\phi\chi\xi+\alpha\phi\xi^2-3)(\chi-\xi)=0,$$

whence the event is constituted as follows—

$$\alpha(\alpha\phi\chi^2-1)(\chi-\xi)+\alpha(\alpha\phi\chi\xi-1)(\chi-\xi)+\alpha(\alpha\phi\xi^2-1)(\chi-\xi)=0;$$

which equation may be resolved into the three equations

$$\alpha(\alpha\phi\chi^2-1)(\chi-\xi)=0,$$
$$\alpha(\alpha\phi\chi\xi-1)(\chi-\xi)=0,$$
$$\alpha(\alpha\phi\xi^2-1)(\chi-\xi)=0,$$

the results being

$$\alpha^2\phi\chi^3+\alpha\xi=\alpha^2\phi\chi^2\xi+\alpha\chi,$$
$$\alpha^2\phi\chi^2\xi+\alpha\xi=\alpha^2\phi\chi\xi^2+\alpha\chi,$$
$$\alpha^2\phi\chi\xi^2+\alpha\xi=\alpha^2\phi\xi^3+\alpha\chi.$$

No one of these phenomena has hitherto been realized; but we are thus led to anticipate their future realization. The unit $\alpha^2\phi\chi\xi^2$, which is characterized by the property of being, together with a unit of water, resoluble into a unit of phosphorous acid and a unit of hydrochloric acid, is (what may be termed) the unit of the monochlorhydrin of phosphorous acid, the unit $\alpha^2\phi\chi^2\xi$ being the dichlorhydrin of the same substance, the existence of which is also indicated to us by a variety of other considerations.

(7) A unit of the trichlorhydrin of glycerin and three units of sulphide of hydrogen are resoluble into three units of hydrochloric acid and a unit of trithioglycerin—

$$\alpha^4\kappa^3\chi^3+3\alpha\theta=3\alpha\chi+\alpha^4\kappa^3\theta^3.$$

This equation may be thus expressed

$$\alpha(\alpha^3\kappa^3\theta^2+\alpha^3\kappa^3\chi\theta+\alpha^3\kappa^3\chi^2-3)(\chi-\theta)=0,$$

the event being constituted as follows—

$$\alpha(\alpha^3\kappa^3\theta^2-1)(\chi-\theta)+\alpha(\alpha^3\kappa^3\chi\theta-1)(\chi-\theta)+\alpha(\alpha^3\kappa^3\chi^2-1)(\chi-\theta)=0;$$

whence we have as the constituents of the event,

$$\alpha(\alpha^3\kappa^3\theta^2-1)(\chi-\theta)=0,$$
$$\alpha(\alpha^3\kappa^3\chi\theta-1)(\chi-\theta)=0,$$
$$\alpha(\alpha^3\kappa^3\chi^2-1)(\chi-\theta)=0,$$

or

$$\alpha^4\kappa^3\theta\chi+\alpha\theta=\alpha\chi+\alpha^4\kappa^3\theta^3,$$
$$\alpha^4\kappa^3\theta\chi^2+\alpha\theta=\alpha\chi+\alpha^4\kappa^3\chi\theta^2,$$
$$\alpha^4\kappa^3\chi^3+\alpha\theta=\alpha\chi+\alpha^4\kappa^3\chi^2\theta.$$

These three equations inform us of the origin and properties of the units of the chlorhydrins of the sulphur-derivatives of glycerin, $\alpha^4\varkappa^3\theta\chi^2$, $\alpha^4\varkappa^3\theta^2\chi$, which substances have been procured by Carius*. In these three events $\alpha$ is constant, and the three events may be referred to one cause, namely the substitution of $\theta$ for $\chi$.

(8) A unit of glycerin and three units of acetic acid are resoluble into three units of water and a unit of triacetin, whence

$$\alpha^4\varkappa^3\xi^3+3\alpha^2\varkappa^2\xi^2=3\alpha\xi+\alpha^7\varkappa^9\xi^6.$$

This equation may be thus expressed

$$-\alpha\xi(\alpha^3\varkappa^3\xi^2+\alpha^4\varkappa^5\xi^3+\alpha^5\varkappa^7\xi^4-3)(\alpha\varkappa^2\xi-1)=0.$$

Proceeding as in the last example, we arrive at the following constituents of the event:—

$$\alpha^4\varkappa^3\xi^3+\alpha^2\varkappa^2\xi^2=\alpha\xi+\alpha^5\varkappa^5\xi^4,$$
$$\alpha^5\varkappa^5\xi^4+\alpha^2\varkappa^2\xi^2=\alpha\xi+\alpha^6\varkappa^7\xi^5,$$
$$\alpha^6\varkappa^7\xi^5+\alpha^2\varkappa^2\xi^2=\alpha\xi+\alpha^7\varkappa^9\xi^6.$$

In these events $\alpha\xi$, the unit of water, is constant. The three events may be referred to one cause, namely the transference of $\alpha\varkappa^2\xi$. These events have been severally realized, the experiments by which Berthelot procured the units of the three acetates of glycerin, monacetin $\alpha^5\varkappa^5\xi^4$, diacetin $\alpha^6\varkappa^7\xi^5$, and triacetin $\alpha^7\varkappa^9\xi^6$, being recorded in these three equations.

(9) Three units of iodide of ethyl and a unit of ammonia are identical with three units of hydriodic acid and a unit of triethylamine,

$$3\alpha^3\varkappa^2\omega+\alpha^2\nu=3\alpha\omega+\alpha^8\varkappa^6\nu.$$

This equation may be thus expressed

$$\alpha(3\omega-\alpha^5\varkappa^4\nu-\alpha^3\varkappa^2\nu-\alpha\nu)(\alpha^2\varkappa^2-1)=0;$$

whence we have the following constituents of this event:—

$$\alpha(\omega-\alpha^5\varkappa^4\nu)(\alpha^2\varkappa^2-1)=0,$$
$$\alpha(\omega-\alpha^3\varkappa^2\nu)(\alpha^2\varkappa^2-1)=0,$$
$$\alpha(\omega-\alpha\nu)(\alpha^2\varkappa^2-1)=0.$$

In these events $\alpha$ is constant; the three events may be referred to a common cause, namely the transference of $\alpha^2\varkappa^2$. The results of these events are given in the following equations:—

$$\alpha^3\varkappa^2\omega+\alpha^6\varkappa^4\nu=\alpha\omega+\alpha^8\varkappa^6\nu,$$
$$\alpha^3\varkappa^2\omega+\alpha^4\varkappa^2\nu=\alpha\omega+\alpha^6\varkappa^4\nu,$$
$$\alpha^3\varkappa^2\omega+\alpha^2\nu\quad=\alpha\omega+\alpha^4\varkappa^2\nu.$$

The three phenomena of which the above event is the aggregate, the realization of

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* Annalen der Chemie, cxxiv. 221.

which is here suggested to us, are the successive stages by which triethylamine has actually been formed in the investigations of HOFMANN.

(10) Lastly, take the familiar phenomenon of the decomposition of water into its elements.

Two units of water are identical with two units of hydrogen and a unit of oxygen, thus

$$1+2\alpha\xi=2\alpha+\xi^2.$$

This equation may be thus expressed,

$$(2\alpha-1-\xi)(\xi-1)=0;$$

whence

$$(\alpha-\xi)(\xi-1)+(\alpha-1)(\xi-1)=0,$$

the constituents of the event being

$$(\alpha-\xi)(\xi-1)=0,$$
$$(\alpha-1)(\xi-1)=0.$$

These two events may be referred to a common cause, namely the transference of $\xi$. Their results are given in the equations

$$\alpha\xi+\xi=\xi^2+\alpha;$$
$$\alpha\xi+1=\xi+\alpha.$$

The element $\xi$ which appears in these phenomena has not been isolated, but its existence is indicated to us as an object of research not here alone, but also by the analyses of numerous other chemical events.

The previous examples are offered to the consideration of the reader, not as illustrations of the general treatment of the subject to be ultimately pursued, but as examples of the most elementary application of the principles of algebraical reasoning, according to the fundamental methods of this calculus, to the analysis of phenomena. By the application to chemical equations of purely formal processes, we elicit from them new and true information as to the chemical occurrences of which they express the results. From this point of view the preceding analyses have a special interest. So far as I am aware they are the very first application which has been made of what can be termed (in any exact sense) algebraical reasoning in chemistry. Such examples might be indefinitely multiplied; but there would be no object in proceeding further in this direction. The method here employed is obviously of but narrow application, being limited to such equations as can be expressed by rational factors (which is an exceptional case), and is superseded by the more general treatment of the subject developed in the following section, to which I will now proceed.

## SECTION V.

(1) We have now constructed, in accordance with the method of this calculus, a symbolical representation of simple chemical events, according to which these events are referred to a definite system of causes, and are conceived of as occurring by means of

operations of which the result is the mutual exchange of the simple weights of which the units of matter are constituted. Here, again, if we would reason correctly, it is essential to discriminate between the operation and the result of the operation. The symbol $(x-a)$ occurring among the factors of a chemical equation is an interpretable symbol, which is defined, not by reference to a hypothetical process, but as the symbol of an operation of which the result is the exchange of the bit of matter resulting from the operation $a$ for the bit of matter resulting from the operation $x$. As, for example, the expression $(\chi-\xi)$ is the symbol of the operation of which the result is the exchange, between two units of matter, of the bit of the matter of oxygen resulting from the operation $\xi$ which weighs 0·715 grm., for the bit of the matter of chlorine resulting from the operation $\chi$ which weighs 1·592 grm., the exchange occurring between two portions of matter the nature of which is not specified, but which severally occupy in the gaseous condition at standard temperature and pressure the space of 1000 cubic centims. But this symbol cannot be interpreted as the symbol of a portion of ponderable matter, for there is no reason to believe the simple weight $w(\xi)$ to be contained in the simple weight $w(\chi)$, and there is no external reality, as far as we are aware, corresponding to the difference $w(\chi)-w(\xi)$. Nor, again, are we to imagine that the process by which this result is attained consists in the actual exchange, in their totality, of these bits of matter the one for the other, as we may exchange a white ball for a red ball. On the contrary, when the unit of hydrochloric acid $\alpha\chi$ passes into the unit of water $\alpha\xi$ by the process of which the final result is the substitution in that unit of $\xi$ for $\chi$, that unit must be regarded as passing by a process of continuous change through every value intermediate between $\alpha\chi$ and $\alpha\xi$, an assumption in perfect accordance with what we know of the gradual character of chemical changes, which are not instantaneous events, but events occurring in definite periods of time. This question first comes under our notice when we pass from the consideration of things and of events to the consideration of the chemical relations of things and the chemical relations of events.

Certain relations of the units of matter are the necessary consequence of the nature of those simple chemical events in which they have their origin. If we consider the symbols of the four units of ponderable matter which appear in a simple event of the second order,

$$A(x-a)(y-b)=0,$$

namely,

$$A\,x\,y,$$
$$A\,a\,y,$$
$$A\,x\,b,$$
$$A\,a\,b,$$

it appears that these symbols are the combinations of the letters $x, y, a, b$, taken two and two, combined with a constant A, and are necessarily derived by substitution the one from the other—$A\,a\,y$ being derived from $A\,x\,y$ by the substitution of $a$ for $x$,

$\mathrm{A}xb$ from $\mathrm{A}xy$ by the substitution of $b$ for $y$, and $\mathrm{A}ab$ from $\mathrm{A}xy$ by the substitution of $a$ for $x$ and of $b$ for $y$. Similarly, in the case of a simple event of the third order,

$$\mathrm{A}(x-a)(y-b)(z-c)=0,$$

we have as the symbols of the units which appear in that event

$$\begin{gathered}\mathrm{A}\,x\,y\,z,\\ \mathrm{A}\,a\,y\,z,\\ \mathrm{A}\,x\,b\,z,\\ \mathrm{A}\,x\,y\,c,\\ \mathrm{A}\,a\,b\,z,\\ \mathrm{A}\,a\,y\,c,\\ \mathrm{A}\,x\,b\,c,\\ \mathrm{A}\,a\,b\,c,\end{gathered}$$

which contain the combinations of the letters $x$, $y$, $z$, $a$, $b$, $c$ taken three and three together, and may severally be regarded as derived from the symbol $\mathrm{A}\,x\,y\,z$ by the substitution in that expression, in all possible ways, of $a$ for $x$, $b$ for $y$, and $c$ for $z$. Similar relations obviously prevail between the symbols of the units of matter which appear in simple events of the fourth or any higher order. These relations are at once perceived on the consideration of the general forms of chemical equations. In special instances they are to a certain extent veiled by the identification of the symbols of some among the simple weights, by the exchanges of which the event occurs, and also by the suppression of the chemical symbol 1. As regards the last point it may be observed that we are always at liberty to replace the chemical symbol 1, where it appears as the symbol of a simple weight, by a special symbol (say the symbol $\varpi$), since this symbol, thus introduced, satisfies all conditions required of the prime factor of a chemical equation. Before interpreting results this symbol is to be put equal to 1. Take, for example, the equation given, Sec. II. (12),

$$\alpha^2\varkappa(\chi-1)^3=0;$$

putting $1=\varpi$ we have as the equation to the event in question the homogeneous equation

$$\alpha^2\varkappa(\chi-\varpi)^3=0,$$

whence

$$(\alpha^2\varkappa)\chi^3+3(\alpha^2\varkappa)\chi\varpi^2=3(\alpha^2\varkappa)\chi^2\varpi+(\alpha^2\varkappa)\varpi^3,$$

the symbols of the units which appear in the event being

$$\begin{gathered}(\alpha^2\varkappa)\chi^3,\\ (\alpha^2\varkappa)\chi^2\varpi,\\ (\alpha^2\varkappa)\chi\varpi^2,\\ (\alpha^2\varkappa)\varpi^3.\end{gathered}$$

So that each symbol may be regarded as derived from the symbol $(\alpha^2\kappa)\chi^3$ by the substitution in all possible ways in that symbol of $\varpi$ for $\chi$. This relation by way of substitution is a universal bond of chemical functions, and prevails among things, among groups of things, and among events. It is desirable to indicate this relation by a name and by a symbol.

(2) Definition:—

Two chemical functions will be said to be congruous to one another in regard to a special substitution if the two functions are of such a nature that they assume the same value when that substitution is effected in them respectively. The value which a function assumes when a special substitution is effected in it will be termed the "residue" of that function in regard to that substitution, and the symbol of the substitution when it appears in this connexion will be termed a "modulus." Further, the symbol $\equiv$ is to be interpreted "is congruous to"; thus, for example, if $f(x)$ be the symbol of a chemical function of the letter $x$, if $x-a$ be the symbol of the modulus, and R the symbol of the residue,

$$f(x)\equiv \mathrm{R},\quad \text{modulus } (x-a).$$

This expression is termed a chemical congruence, and is to be read $f(x)$ "is congruous to" R for the modulus $(x-a)$.

Similarly we have

$$f(x, y, z, \ldots)\equiv \mathrm{R},\quad \text{mod } (x-a) \text{ mod } (y-b) \text{ mod } (z-c)\ldots,$$

where R is the value which $f(x, y, z, \ldots)$ assumes when in that function $a$ is substituted for $x$, $b$ for $y$, $c$ for $z$ . . .

(3) I proceed to determine the conditions satisfied by functions congruous to a given residue for a given modulus.

Let
$$f(x)\equiv \mathrm{R},\quad \text{mod } (x-a);$$

for $x$ in $f(x)$ we will write $a+x-a$: we have then

$$f(x)=f(a+\overline{x-a}).$$

Now since $f(x)$ is a rational and integral function of $x$ (for there are no other chemical functions), $f(a+\overline{x-a})$ is a rational and integral function of $(x-a)$. Putting then

$$\mathrm{A}_0+\mathrm{A}_1(x-a)+\mathrm{A}_2(x-a)^2+\ldots\ldots \mathrm{A}_n(x-a)^n$$

as the general expression for such a function of $(x-a)$, where $\mathrm{A}_0$, $\mathrm{A}_1$, $\mathrm{A}_2$, . . . $\mathrm{A}_n$ are functions of $a$ and free from $x$, and where $n$ is a positive integer, we have

$$f(a+\overline{x-a})=\mathrm{A}_0+\mathrm{A}_1(x-a)+\mathrm{A}_2(x-a)^2+\ldots \mathrm{A}_n(x-a)^n.$$

Now it has been demonstrated (Sec. I. (4)) that every continued product of two or more chemical factors of the form $(x-a)(y-b)\ldots$ is necessarily equal to zero.

Hence

$$A_2(x-a)^2=0,$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$A_n(x-a)^n=0,$$

and

$$f(a+\overline{x-a})=A_0+A_1(x-a).$$

Since this last equation is always true, it is true when $x=a$; putting $x=a$,

$$f(a)=A_0;$$

and also since the residue R is the value of $f(x)$, when $x=a$,

$$A_0=R,$$

and

$$f(x)=f(a)+A_1(x-a).$$

To determine $A_1$ we have

$$A_1=\frac{f(x)-f(a)}{x-a}.$$

This equation again being always true is true when $x=a$; but the limit of the value of

$$\frac{f(x)-f(a)}{x-a},$$

when $x=a$, is $f_1(a)$, where $f_1(a)$ is the first derived function of $f(x)$, that is $\frac{df(x)}{dx}$, in which $a$ is substituted for $x$. We have therefore for the value of $A_1$,

$$A_1=f_1(a),$$

and

$$f(x)=f(a)+f_1(a)(x-a).$$

Similarly, in the case of the congruence

$$f(x,y)\equiv f(a,b), \ \text{mod}\,(x-a)\,\text{mod}\,(y-b),$$

we have, regarding $y$ as a constant and developing by the above theorem,

$$f(x,y)=f(a,y)+f_1(a,y)(x-a),$$

and also

$$f(a,y)=f(a,b)+f_1(a,b)(y-b),$$
$$f_1(a,y)=f_1(a,b)+f_{1.1}(a,b)(y-b);$$

whence, substituting these values for $f(a,y)$ and $f_1(a,y)$, we have

$$f(x,y)=f(a,b)+f_{1.0}(a,b)(x-a)+f_{0.1}(a,b)(y-b)+f_{1.1}(a,b)(x-a)(y-b),$$

and

$$f(x,y)=f(a,b)+f_{1.0}(a,b)(x-a)+f_{0.1}(a,b)(y-b).$$

Similarly, considering the congruence

$$f(x, y, z) \equiv f(a, b, c), \quad \bmod (x-a) \bmod (y-b) \bmod (z-c),$$

we have

$$f(x, y, z) = f(a, b, c) + f_{1.0.0}(a, b, c)(x-a) + f_{0.1.0}(a, b, c)(y-b) + f_{0.0.1}(a, b, c)(z-c),$$

and so on, a similar relation holding good in the general case

$$f(x,y,z,v,w,\ldots) \equiv f(a,b,c,d,e,\ldots), \bmod(x-a) \bmod(y-b) \bmod(z-c) \bmod(v-d) \bmod(w-e)\ldots$$

(4) The symbol $\equiv$, which is here adopted as the symbol of "chemical congruence," is used in mathematical investigations of the properties of numbers as the symbol of numerical congruence. Thus the expression $x \equiv a, \bmod p$ indicates to us that the integral number $x$ satisfies the condition given in the equation $x = a + py$, where $a$, $p$, $y$ are integers as well as $x$, so that the difference $x-a$ is divisible without remainder by the modulus $p$. Now the simplest form of a chemical congruence, $f(x) \equiv f(a)$, mod $(x-a)$, is, in the most exact sense, an "algebraical congruence," for $f(x)$ satisfies the condition $f(x) = f(a) + \mathrm{Q}(x-a)$, where $f(x)$, $f(a)$, and Q are respectively rational and integral functions of $x$ and $a$, so that $f(x) - f(a)$ is divisible without remainder by the modulus $x-a$. In the case of the chemical congruence to two moduli, such as $f(x, y) \equiv f(a, b)$, mod $(x-a)$ mod $(y-b)$, where $f(x, y)$ satisfies the condition $f(x, y) = f(a, b) + \mathrm{A}_1(x-a) + \mathrm{B}_1(y-b)$, an analogous condition prevails. In this case if we divide the difference $f(x, y) - f(a, b)$ by the modulus $(x-a)$ we have as the remainder of the division $f(a, y) - f(a, b)$, which is divisible without remainder by the modulus $y-b$. It hence appears that a "chemical congruence" for two or more moduli is but a wider kind of numerical congruence, being a comprehensive form in which all such "congruences" are included. The same fundamental idea of congruity is applicable in either case; and as no confusion is likely to arise from the use of the symbol $\equiv$ as the symbol of the chemical relation referred to, which is, indeed, the fundamental relation of the science of chemistry, I have not hesitated to adopt it.

(5) From the various illustrations already given of "simple chemical events," taken together with the explanation given in Section II. of the aggregation of events, the following definition of the "theoretical analysis" of a compound chemical event occurring by any number of substitutions will be readily appreciated.

Definition:—The theoretical analysis of a chemical event occurring by any number of specified substitutions, namely, by the substitution of $a$ for $x$, $b$ for $y$, $c$ for $z$, and so on, will be here said to be effected when all the different chemical events, occurring in any way whatever by these substitutions, are enumerated, the aggregate of which constitutes the event in question; and the real analysis of such an event will be said to be completely effected when all these events are severally realized as independent phenomena, and will be said to be partially effected when two or more of such events or aggregates of such events are so realized.

This analytical problem is presented to us in every chemical congruence.

The congruence

$$f(x, y, z \ldots) \equiv f(a, b, c \ldots), \quad \bmod (x-a) \bmod (y-b) \bmod (z-c) \ldots$$

asserts that the residue $f(a, b, c \ldots)$ is derived from the function $f(x, y, z \ldots)$ by the substitutions specified in the modules. The problem now to be considered is in what way these substitutions are effected. The solution of this problem is afforded by means of the following theorem.

The demonstration of TAYLOR'S theorem, in which it is assumed that $f(x)$ may be developed in a series of ascending powers of $x$ with integral indices, is entirely independent of the interpretation of the symbols, and is recognized as dependent solely upon those formal properties of symbols expressed in the equations $xy=yx$, $x(y+z)=xy+xz$, known as the commutative and distributive laws. Now in the first part of this Calculus (Part I. Sec. II. (5), (6)) it has been fully demonstrated that the symbols of chemical operations, with the interpretation there assigned to them, satisfy these fundamental conditions. Precisely, therefore, as we are enabled through these properties of those symbols to work with them according to the processes of elementary algebra, so we are equally justified in applying to them the principles of the differential Calculus regarded from this point of view. I shall now prove that if $f(x)$ satisfies the condition given in the congruence

$$f(x) \equiv f(a), \quad \bmod (x-a),$$

$$f(x)=f(a)+f_1(a)(x-a)+\frac{1}{1.2}f_2(a)(x-a)^2+\frac{1}{1.2.3}f_3(a)(x-a)^3+\ldots+\frac{1}{\underline{|n}}f_n(a)(x-a)^n,$$

where $f_1(a), f_2(a), f_3(a) \ldots f_n(a)$ are the first, second, third, and $n$th derived functions of $f(x)$ in which $a$ is substituted for $x$.

For let $f(x)$ be any function of $x$, and in that function let $x$ undergo a variation of such a nature that $x$ becomes $x+\Delta x$; we then have, by TAYLOR'S theorem,

$$f(x+\Delta x)=f(x)+f_1(x)\Delta x+\frac{1}{1.2}f_2(x)\Delta x^2+\frac{1}{1.2.3}f_3(x)\Delta x^3+\ldots+\frac{1}{\underline{|n}}f_n(x)\Delta x^n.$$

Now let $\Delta x$ be that variation which $x$ undergoes when $a$ is substituted for $x$; we have then $x+\Delta x=a$, $\Delta x=a-x$, and $f(x+\Delta x)=f(a)$. Substituting in the above development $f(a)$ for $f(x+\Delta x)$ and $a-x$ for $\Delta x$, we have

$$f(a)=f(x)+f_1(x)(a-x)+\frac{1}{1.2}f_2(x)(a-x)^2+\frac{1}{1.2.3}f_3(x)(a-x)^3+\ldots+\frac{1}{\underline{|n}}f_n(x)(a-x)^n,$$

whence, changing $a$ into $x$ and $x$ into $a$,

$$f(x)=f(a)+f_1(a)(x-a)+\frac{1}{1.2}f_2(a)(x-a)^2+\frac{1}{1.2.3}f_3(a)(x-a)^3+\ldots+\frac{1}{\underline{|n}}f_n(a)(x-a)^n.$$

Now among the properties of chemical functions demonstrated in Sec. I. it has been shown that every expression of the form $(x-a)^n$, when $n>1$, is necessarily equal to zero.

We may therefore at once resolve the above equation into the following system of equations:—

$$\text{I.}\quad f(x)-f(a)-f_1(a)(x-a)=0.$$

$$\text{II.}\quad \frac{1}{1.2}f_2(a)(x-a)^2=0.$$

$$\text{III.}\quad \frac{1}{1.2.3}f_3(a)(x-a)^3=0.$$

. . . . . . . . .

. . . . . . . . .

$$\frac{1}{\underline{|n}}f_n(a)(x-a)^n=0.$$

The first of these equations is that which has been already given as indicating the condition by which $f(x)$ is related to the residue $f(a)$. The aggregate of the succeeding equations is identical with that equation; and the equations separately considered inform us of the successive steps by which the result indicated in equation I.,

$$f(x)=f(a)+f_1(a)(x-a),$$

is attained.

This method of development is applicable to every chemical function; but if $f(x)$ be a chemical equation of the form $f(x)=0$, it follows from the property of chemical equations demonstrated (Section I. (8)) that $f(a)$ is also a chemical equation; whence

$$f(a)=0,$$
$$f_1(a)(x-a)=0.$$

It is to be observed that in this development of $f(x)$ in ascending powers of the modulus $(x-a)$, the symbols of the units of matter, which appear in the several events of which $f(x)$ is constituted, are resolved into their components according to the classification of those components given in Section III. (3), namely, as "variables," "values," and "constants,"—the "variable" being symbolized as $x$, the "value" of that "variable" as $a$, and the "constants" being given in the coefficients

$$f_1(a),\ \frac{1}{1.2}f_2(a),\ \frac{1}{1.2.3}f_3(a),\ \ldots\ \frac{1}{\underline{|n}}f_n(a).$$

This classification of the components of the units of matter thus satisfies the conditions required of an accurate method of classification, being both exclusive and exhaustive.

A chemical function of two variables $f(x, y)$ satisfying the condition

$$f(x,y)\equiv f(a,b),\quad \text{mod}\ (x-a)\ \text{mod}\ (y-b)$$

may be developed on precisely the same principles.

Using a similar notation to that previously employed, let

$$f_1\ (x,y)=\frac{df(x,y)}{dx},$$

$$f_{0.1}(x,y)=\frac{df(x,y)}{dy},$$

$$f_{1.1}(x,y)=\frac{d^2f(x,y)}{dx.dy},$$

$$f_2\ (x,y)=\frac{d^2f(x,y)}{dx^2},$$

$$f_{0.2}(x,y)=\frac{d^2f(x,y)}{dy^2},$$

and generally

$$f_{m.n}(x,y)=\frac{d^{m+n}f(x,y)}{d^mx.d^ny},$$

and let

$$f_{m.n}(a,b)=f_{m.n}(x,y),$$

in which $a$ is substituted for $x$, and $b$ for $y$.

Now in $f(x,y)$ let $x$ and $y$ respectively vary, so that $x$ becomes $x+\Delta x$ and $y$ becomes $y+\Delta y$; we then have, by TAYLOR's theorem,

$$\begin{aligned}f(x+\Delta x,\ y+\Delta y)&=f(x,y)+f_1(x,y)\Delta x+f_{0.1}(x,y)\Delta y+\frac{1}{1.2}f_2(x,y)\Delta x^2\\&+f_{1.1}(x,y)\Delta x\Delta y+\frac{1}{1.2}f_{0.2}(x,y)\Delta y^2+\frac{1}{\lfloor\underline{3}}f_3(x,y)\Delta x^3+\frac{1}{\lfloor\underline{2}}f_{2.1}(x,y)\Delta x^2\Delta y\\&+\frac{1}{\lfloor\underline{2}}f_{1.2}(x,y)\Delta x\Delta y^2+\frac{1}{\lfloor\underline{3}}f_{0.3}(x,y)\Delta y^3\ldots+\frac{1}{\lfloor\underline{m}\lfloor\underline{n}}f_{m.n}(x,y)\Delta x^m\Delta y^n,\end{aligned}$$

whence, assuming as before $\Delta x$ to be the variation which $x$ undergoes when $x$ is substuted for $a$, and $\Delta y$ the variation which $y$ undergoes when $y$ is substituted for $b$, we have

$$\Delta x=a-x,$$
$$\Delta y=b-y.$$

Substituting these values in the above development for $\Delta x$ and $\Delta y$, and again changing in the result $a$ into $x$ and $x$ into $a$, and $b$ into $y$ and $y$ into $b$, we have

$$\begin{aligned}f(x,y)&=f(a,b)+f_1(a,b)(x-a)+f_{0.1}(a,b)(y-b)+\frac{1}{\lfloor\underline{2}}f_2(a,b)(x-a)^2+f_{1.1}(a,b)(x-a)(y-b)\\&+\frac{1}{\lfloor\underline{2}}f_{0.2}(a,b)(y-b)^2+\frac{1}{\lfloor\underline{3}}f_3(a,b)(x-a)^3+\frac{1}{\lfloor\underline{2}}f_{2.1}(a,b)(x-a)^2(y-b)\\&+\frac{1}{\lfloor\underline{2}}f_{1.2}(a,b)(x-a)(y-b)^2+\frac{1}{\lfloor\underline{3}}f_{0.3}(a,b)(y-b)^3\ldots+\frac{1}{\lfloor\underline{m}\lfloor\underline{n}}f_{m.n}(a,b)(x-a)^m(y-b)^n.\end{aligned}$$

On the principles laid down in the last case, this equation may be resolved into the following system of equations, by which it is adequately represented:—

$$f(x,y)=f(a,b)+f_1(a,b)(x-a)+f_{0.1}(a,b)(y-b),$$

$$\frac{1}{\underline{|2}}f_2(a,b)(x-a)^2=0,$$

$$f_{1.1}(a,b)(x-a)(y-b)=0,$$

$$\frac{1}{\underline{|2}}f_{0.2}(a,b)(y-b)^2=0,$$

$$\frac{1}{\underline{|3}}f_3(a,b)(x-a)^3=0,$$

$$\frac{1}{\underline{|2}}f_{2.1}(a,b)(x-a)^2(y-b)=0,$$

$$\frac{1}{\underline{|2}}f_{1.2}(a,b)(x-a)(y-b)^2=0,$$

$$\frac{1}{\underline{|3}}f_{0.3}(a,b)(y-b)^3=0,$$

. . . . . . . . . . . . . .

$$\frac{1}{\underline{|m}\,\underline{|n}}f_{m.n}(a,b)(x-a)^m(y-b)^n=0.$$

If $f(x,y)$ be a chemical equation, so that

$$f(x,y)=0,$$

$f(a,b)$ is also a chemical equation, so that

$$f(a,b)=0;$$

and, moreover, observing that

$$f_1\ (a,b)(x-a)=f(x,b)-f(a,b),$$
$$f_{0.1}(a,b)(y-b)=f(a,y)-f(a,b),$$

$f(x,b)$ and $f(a,y)$ also being chemical equations, we have

$$f_1\ (a,b)(x-a)=0,$$
$$f_{0.1}(a,b)(y-b)=0.$$

In the two last equations, taken together with the equation $\frac{1}{\underline{|2}}f_2(a,b)(x-a)^2=0$ and the succeeding equations of the system, the theoretical analysis of the event $f(x,y)=0$ is effected, since these equations are collectively identical with the equation $f(x,y)=0$, and in all respects adequately represent that equation.

By the application of these principles we arrive at the following rule for the development of a chemical function $f(x,y,z,v,w,\ldots)$, which satisfies the congruence

$$f(x,y,z,v,w,\ldots)\equiv f(a,b,c,d,e,\ldots),\ \text{mod}\,(x-a)\,\text{mod}\,(y-b)\,\text{mod}\,(z-c)\,\text{mod}\,(v-d)\ldots.$$

Develop by TAYLOR'S theorem $f(x+\Delta x,\ y+\Delta y,\ z+\Delta z,\ v+\Delta v,\ldots)$ in ascending powers of $\Delta x$, $\Delta y$, $\Delta z$, $\Delta v$, $\Delta w$, . . . Substitute in the development thus effected for

$\Delta x$, $\Delta y$, $\Delta z$, $\Delta v, \ldots$ respectively $a-x$, $b-y$, $c-z$, $d-v, \ldots$, and in the result change $a$ into $x$ and $x$ into $a$, $b$ into $y$ and $y$ into $b$, $c$ into $z$ and $z$ into $c$, $d$ into $v$ and $v$ into $d$, and so on. We hence arrive at the development of $f(x, y, z, v, \ldots)$ in ascending powers of the moduli $x-a$, $y-b$, $z-c$, $v-d \ldots$. Proceeding as before we have

$$\begin{aligned} f(x, y, z, v, w, \ldots) = {} & f(a, b, c, d, e, \ldots) + f_1(a, b, c, d, e, \ldots)(x-a) \\ & + f_{0.1}(a, b, c, d, e, \ldots)(y-b) + f_{0.0.1}(a, b, c, d, e, \ldots)(z-c) \\ & + f_{0.0.0.1}(a, b, c, d, e, \ldots)(v-d) + \&c., \end{aligned}$$

together with a system of equations of the form

$$\frac{1}{\underline{|m}\,\underline{|n}\,\underline{|p}\,\underline{|q}\ldots} f_{m.n.p.q}(a, b, c, d, \ldots)(x-a)^m(y-b)^n(z-c)^p(v-d)^q \ldots = 0.$$

Reasoning as before, if $f(x, y, z, v, \ldots)$ be a chemical equation, we have

$$\begin{aligned} & f(x, y, z, v, \ldots) = 0, \\ & f(a, b, c, d, \ldots) = 0. \\ & f_1(a, b, c, d, \ldots)(x-a) = 0, \\ & f_{0.1}(a, b, c, d, \ldots)(y-b) = 0, \\ & f_{0.0.1}(a, b, c, d, \ldots)(z-c) = 0, \\ & f_{0.0.0.1}(a, b, c, d, \ldots)(v-d) = 0. \end{aligned}$$

In these last equations, taken together with the equations previously referred to of the second and higher orders, the theoretical analysis of the event $f(x, y, z, v, \ldots) = 0$ is effected, these equations collectively representing, for all purposes, the equation $f(x, y, z, v, \ldots) = 0$.

As a necessary preliminary to the analysis of a compound chemical event, the substitutions must be specified in reference to which the analysis is to be effected. We can then proceed by means of the preceding theorem to resolve the event $f(x, y, z, \ldots) = 0$ into its constituents, which consist essentially of two groups, the event $f(a, b, c, \ldots) = 0$, which does not occur by the substitutions in question, and the system of simple events enumerated in the various terms of the development, occurring in all possible ways by these substitutions, an analysis which, as we have said, is absolutely exhaustive.

(6) From these considerations we arrive at a more exact definition than has hitherto been open to us of a normal chemical equation, and of the position which it occupies in the general algebraical system. We have seen that in the case of any chemical equation $f(x, y, z, \ldots) = 0$,

$$\begin{aligned} & f(a, b, c, d, \ldots) = 0, \\ & f_1(a, b, c, d, \ldots)(x-a) = 0, \\ & f_{0.1}(a, b, c, d, \ldots)(y-b) = 0, \\ & f_{0.0.1}(a, b, c, d, \ldots)(z-c) = 0, \\ & \ldots\ldots\ldots\ldots \end{aligned}$$

Now, since these equations are always true, they are true when $a=1$, $b=1$, $c=1$; whence

$$\begin{aligned} &f(1, 1, 1, 1\ldots)=0, \\ &f_1(1, 1, 1, 1\ldots)(x-1)=0. \\ &f_{0.1}(1, 1, 1, 1\ldots)(y-1)=0, \\ &f_{0.0.1}(1, 1, 1, 1\ldots)(z-1)=0. \\ &\ldots\ldots\ldots\ldots \end{aligned}$$

Hence it follows that if in any chemical equation $u=0$ the prime factors by which the equation is expressed be severally and simultaneously put equal to 1 (which is the only numerical value of chemical symbols), that equation vanishes, and also if we differentiate that equation once in regard to any one (and every one) of these factors, and in the result of that differentiation put all the prime factors severally and simultaneously equal to 1, that differential coefficient also vanishes*. A chemical equation, therefore, may be defined as an equation which possesses these properties.

(7) Further, these considerations supply us with a general and comprehensive theory of the mode of occurrence of chemical events. It has been shown in the first part of this Calculus that in every chemical equation the symbols of the units of matter are expressed by the smallest possible number of prime factors when expressed by the factors $\alpha$, $\xi$, $\theta$, $\chi$, $\beta$, $\omega$, $\nu$, $\phi$, ... So that every chemical equation regarded as a member of the general system is of the form

$$f(\alpha, \xi, \theta, \chi, \beta, \omega, \nu, \phi, \ldots)=0.$$

Now every such equation necessarily vanishes when the prime factors $\alpha, \xi, \theta, \chi, \ldots$ are severally and simultaneously put equal to 1, and also admits of being developed in ascending powers of the moduli $(\alpha-1), (\xi-1), (\theta-1), (\chi-1), \ldots$ Hence every chemical event, without exception, may be considered to occur by the transferences of the simple weights $w(\alpha)$, $w(\xi)$, $w(\theta)$, $w(\chi)$, . . ., and admits of being resolved into a set of "constituent simple events" severally occurring in various ways by these same transferences; and if we are to refer all chemical events to one set of causes, so as to bring them under one law, these transferences are the only set of causes to which these phenomena can be referred. Thus, as we are led to contemplate the resolution of the units of matter into the "system of simple weights" $w(\alpha)$, $w(\xi)$, $w(\theta)$, $w(\chi)$, . . . as the ultimate

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* This latter property of a chemical equation has already been demonstrated in Part I. Section V., and forms indeed the basis of this Calculus. It here reappears in another form. The condition $f(1, 1\ldots)=0$ is (as is obvious) a necessary property of every chemical equation when brought to the normal form in the way indicated in Section I. (3). But it may be noticed that the analysis of a chemical event is quite independent of the prior reduction to this form, which is effected in the course of the development itself. Take, for example, the equation given in I. Section IV. (1), $xy=x+y$, the fundamental chemical equation. The equation in this form does not inform us of the process by which the transformation in question is effected. But putting $f(x,y)=xy-x-y$ and developing we have $f(1, 1)=-1$, $f_{1.0}(1, 1)=0$, $f_{0.1}(1, 1)=0$, $f_{1.1}(1, 1)=1$, and $f(x, y)=-1+(x-1)(y-1)$ or $f(x, y)+1=(x-1)(y-1)$, the equation $f(x, y)+1=0$ being the equation $xy=x+y$ reduced to the normal form, namely $1+xy=x+y$.

limit of the analysis of those units, so here another and not altogether dissimilar problem is suggested to us—namely, the actual analysis of all chemical events into a system of simple constituent events occurring in various ways by the transferences to and fro among the units of matter and space of these same "simple weights." We are far indeed from this ultimate ideal goal, but may yet recognize it on the far horizon as the limit of our speculations.

(8) We are thus led to regard many events of an apparently simple character as constituted of numerous other events, some realizable, others not, the concurrence of which results in those events. To some, even when demonstrated, this may appear a complex view. But the complexity is not real. It is in truth the simplest possible doctrine. We may compare the aggregates of simple events of which compound events are constituted to the aggregate of the repeated blows of the hammer, by which (each falling with a certain force and in a certain requisite direction) two or more pieces of iron are welded together and shaped to a determinate form. What is here effected is to specify the kind of blows which are required and the number of blows of each kind. Now it may be asked, since every chemical event is here referred to one set of causes, and regarded as occurring by the transferences of the "simple weights" $\alpha, \xi, \chi, \ldots,$ How are we to interpret such expressions as $(x-a)$, $(xy-ab)$, when they occur among the factors of chemical equations? Such expressions apparently involve a change in our point of view; for $(x-a)$ is not the symbol of the "transference" of $x$, but of the substitution of one "simple weight," $a$, for another, $x$, and $(xy-ab)$ is not the symbol of any substitution of one simple weight for another, but is the symbol of the substitution of a compound weight $ab$ for a compound weight $xy$. To this it may be replied that we are here considering results; that the expression $(x-a)$ means the occurrence of a phenomenon of which the result is the substitution of $a$ for $x$. But it does not inform us of any particular way in which this result is attained. And, in fact, this result may with equal justice be considered as the aggregate of the two transferences $(x-1)-(a-1)$ as a single substitution. Such an expression, therefore, as $(x-a)(y-b)$ is always to be regarded as an abbreviated form of expression for the aggregate

$$(x-1)(y-1)-(x-1)(b-1)-(a-1)(y-1)+(a-1)(b-1).$$

Similarly, developing the expression $(xy-ab)$, we have

$$xy-ab=(x-1)+(y-1)+(x-1)(y-1)-(a-1)-(b-1)-(a-1)(b-1),$$

so that the event symbolized (for example) as $(xy-ab)(z-1)$ is to be regarded as an abbreviated expression for the aggregate of events,

$$(x-1)(z-1)+(y-1)(z-1)+(x-1)(y-1)(z-1)-(a-1)(z-1)-(b-1)(z-1)$$
$$-(a-1)(b-1)(z-1).$$

It is my intention to lay before the Society, in a further communication, a set of examples illustrative of the application of the principles of this Calculus to the discussion of special chemical events.